No Worksheet

For

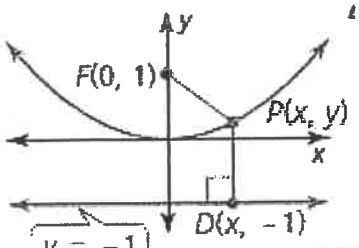
This Lesson

=)

College Algebra – Chapter 10
Lesson 2

Use the distance formula to write an equation of the parabola.

1.



$\sqrt{(x-0)^2+(y-1)^2} = \sqrt{(x-x)^2+(y+1)^2}$

$\left(\sqrt{x^2+(y-1)^2}\right)^2 = \left(\sqrt{(y+1)^2}\right)^2$

$x^2+(y-1)^2 = (y+1)^2$

$x^2+y^2-2y+1 = y^2+2y+1$

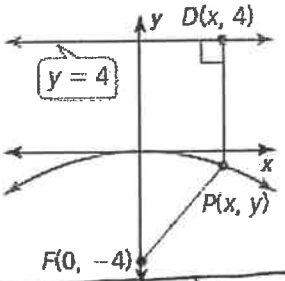
$x^2+y^2-2y+1 = y^2+2y+1$

$-y^2+2y-1 \quad -y^2+2y-1$

$\frac{x^2}{4} = \frac{4y}{4}$

$y = \frac{1}{4}x^2$

2.



$\sqrt{(x-0)^2+(y+4)^2} = \sqrt{(x-x)^2+(y-4)^2}$

$\sqrt{x^2+(y+4)^2} = \sqrt{(y-4)^2}$

$x^2+(y+4)(y+4) = (y-4)(y-4)$

$x^2+y^2+8y+16 = y^2-8y+16$

$x^2+y^2+8y+16 = y^2-8y+16$

$-y^2-8y-16 \quad -y^2-8y-16$

$\frac{x^2}{-16} = \frac{-16y}{-16}$

$y = \frac{-1}{16}x^2$

3. focus: (0, -2); directrix: y = 2

$\sqrt{(x-0)^2+(y+2)^2} = \sqrt{(x-x)^2+(y-2)^2}$

$\sqrt{x^2+(y+2)^2} = \sqrt{(y-2)^2}$

$x^2+(y+2)^2 = (y-2)^2$

$x^2+y^2+4y+4 = y^2-4y+4$

$-y^2-4y-4 \quad -y^2-4y-4$

$\frac{x^2}{-8} = \frac{-8y}{-8}$

$y = -\frac{1}{8}x^2$

4. Directrix: y = 7; focus: (0, -7)

(x, 7)

$\sqrt{(x-x)^2+(y-7)^2} = \sqrt{(x-0)^2+(y+7)^2}$

$\sqrt{(y-7)^2} = \sqrt{x^2+(y+7)^2}$

$(y-7)^2 = x^2+(y+7)^2$

$y^2-14y+49 = x^2+y^2+14y+49$

$-y^2-14y-49 \quad -y^2-14y-49$

$\frac{-28y}{-28} = \frac{x^2}{-28}$

$y = -\frac{1}{28}x^2$

5. vertex: (0, 0); directrix: y = -6

directrix → y = -6, so Vertical Axis

$y = \frac{1}{4p}x^2$

$y = -p$

$y = -6$, so $p = 6$

$y = \frac{1}{4(6)}x^2$

$y = \frac{1}{24}x^2$

6. Vertex: (0, 0); focus: (0, 5)

Focus (0, 5) → (0, p)

$p = 5$

$y = \frac{1}{4p}x^2$ (5 is in "y")

$y = \frac{1}{4(5)}x^2$

$y = \frac{1}{20}x^2$

7. vertex: (0, 0); focus: (0, -10)

Focus (0, -10) → (0, p)

$p = -10$

$y = \frac{1}{4p}x^2$ (-10 is "y")

$y = \frac{1}{4(-10)}x^2$

$y = -\frac{1}{40}x^2$

8. Vertex: (0, 0); directrix: y = -9

Directrix → y = -9 → y = -p

$p = 9$

$y = \frac{1}{4p}x^2$ (Directrix is y =)

$y = \frac{1}{4(9)}x^2$

$y = \frac{1}{36}x^2$

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Identify the focus, directrix, and axis of symmetry of the parabola.

9. $y = \frac{1}{8}x^2 \rightarrow y = \frac{1}{4(2)}x^2$

$p = 2$

Focus: $(0, 2)$

Directrix: $y = -2$

Ax of S: $x = 0$

10. $y = \frac{1}{12}x^2 \rightarrow y = \frac{1}{4(-3)}x^2$

$p = -3$

Focus: $(0, -3)$

Directrix: $y = 3$

Ax of S: $x = 0$

11. $x = \frac{1}{12}y^2 \rightarrow x = \frac{1}{4(-3)}y^2$

$p = -3$

Focus: $(-3, 0)$

Directrix: $x = 3$

Ax of S: $y = 0$

12. $x = \frac{1}{24}y^2 \rightarrow x = \frac{1}{4(6)}y^2$

$p = 6$

Focus: $(6, 0)$

Directrix: $x = -6$

Ax of S: $y = 0$

13. $\frac{y^2}{16} = \frac{16x}{16}$

$x = \frac{1}{16}y^2 \rightarrow x = \frac{1}{4(4)}y^2$

$p = 4$

Focus: $(4, 0)$

Directrix: $x = -4$

Ax of S: $y = 0$

14. $\frac{-x^2}{48} = \frac{48y}{48}$

$y = -\frac{1}{48}x^2 \rightarrow y = \frac{1}{4(-12)}x^2$

$p = -12$

Focus: $(0, -12)$

Directrix: $y = 12$

Ax of S: $x = 0$

15. $6x^2 + 3y = 0$
$-3y \quad -3y$

$\frac{-3y}{-3} = \frac{6x^2}{-3}$

$y = -2x^2 \rightarrow y = \frac{1}{4(-\frac{1}{8})}x^2$

$p = -\frac{1}{8}$

Focus: $(0, -\frac{1}{8})$

Directrix: $y = \frac{1}{8}$

Ax of S: $x = 0$

16. $8x^2 - y = 0$
$+y \quad +y$

$y = 8x^2 \rightarrow y = \frac{1}{4(\frac{1}{32})}x^2$

$p = \frac{1}{32}$

Focus: $(0, \frac{1}{32})$

Directrix: $y = -\frac{1}{32}$

Ax of S: $x = 0$

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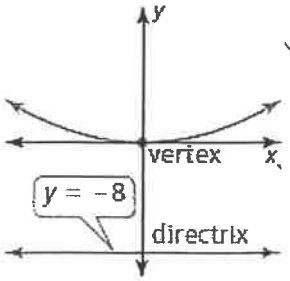
Lesson 2

Write an equation of the parabola shown.

17.



Directrix → $y = -8$

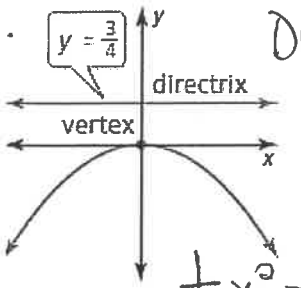
$p = 8$

$y = \frac{1}{4p}x^2$

$y = \frac{1}{4(8)}x^2$

$y = \frac{1}{32}x^2$

18.



Directrix: $y = \frac{3}{4}$

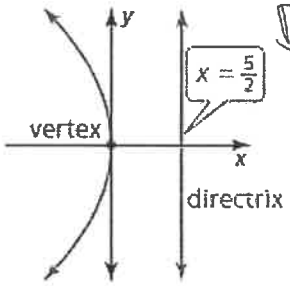
$p = -\frac{3}{4}$

$y = \frac{1}{4(-\frac{3}{4})}x^2$

$y = \frac{1}{-12/4}x^2 \Rightarrow -\frac{1}{3}x^2$

$y = -\frac{1}{3}x^2$

19.



Directrix: $x = \frac{5}{2}$

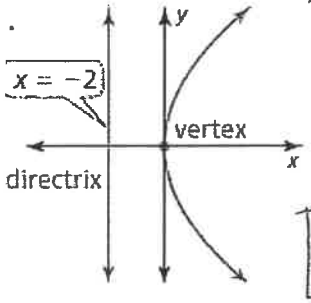
$p = -\frac{5}{2}$

$x = \frac{1}{4(-\frac{5}{2})}y^2$

$x = -\frac{1}{20/2}y^2 \rightarrow -\frac{1}{10}y^2$

$x = -\frac{1}{10}y^2$

20.



Directrix: $x = -2$

$p = 2$

$x = \frac{1}{4(2)}y^2$

$x = \frac{1}{8}y^2$

Write an equation of the parabola with the given characteristics.

21. Focus: (3, 0); directrix: x = -3

$p = 3$

$x = \frac{1}{4(3)}y^2$

$x = \frac{1}{12}y^2$

22. Focus: (2/3, 0); directrix: x = -2/3

$p = \frac{2}{3}$

$x = \frac{1}{4(\frac{2}{3})}y^2 \Rightarrow \frac{1}{8/3}y^2$

$\Rightarrow 1 \cdot \frac{3}{8}y^2 \Rightarrow \frac{3}{8}y^2$

$x = \frac{3}{8}y^2$

23. directrix: x = -10; vertex: (0, 0)

$p = 10$

$x = \frac{1}{4(10)}y^2$

$x = \frac{1}{40}y^2$

24. Directrix: y = 8/3; vertex: (0, 0)

$p = -\frac{2}{3}$

$y = \frac{1}{4(-2/3)}x^2 \Rightarrow -\frac{1}{32/3}x^2$

$\Rightarrow 1 \cdot -\frac{3}{32}x^2 = -\frac{3}{32}x^2$

$y = -\frac{3}{32}x^2$

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25. focus: (0, -5/3); directrix: y = 5/3

$p = -\frac{5}{3}$

$y = \frac{1}{4(-\frac{5}{3})}x^2 \Rightarrow -\frac{1}{\frac{20}{3}}x^2$

$\Rightarrow -\frac{3}{20}x^2 \Rightarrow -\frac{3}{20}x^2$

$y = -\frac{3}{20}x^2$

26. Focus: (0, 5/4); directrix: y = -5/4

$p = \frac{5}{4}$

$y = \frac{1}{4(\frac{5}{4})}x^2 \Rightarrow \frac{1}{5}x^2$

$y = \frac{1}{5}x^2$

27. focus: (0. 6/7); vertex: (0, 0)

$p = \frac{6}{7}$

$y = \frac{1}{4(\frac{6}{7})}x^2 \Rightarrow \frac{1}{\frac{24}{7}}x^2$

$\Rightarrow \frac{7}{24}x^2 = \frac{7}{24}x^2$

$y = \frac{7}{24}x^2$

28. Focus: (-4/5, 0); vertex: (0, 0)

$p = -\frac{4}{5}$

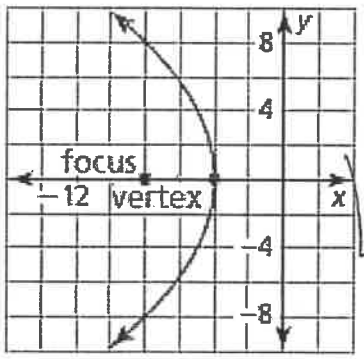
$x = \frac{1}{4(-\frac{4}{5})}y^2 \Rightarrow -\frac{1}{\frac{16}{5}}y^2$

$\Rightarrow -\frac{5}{16}y^2 \Rightarrow -\frac{5}{16}y^2$

$x = -\frac{5}{16}y^2$

Write an equation of the parabola shown.

29.



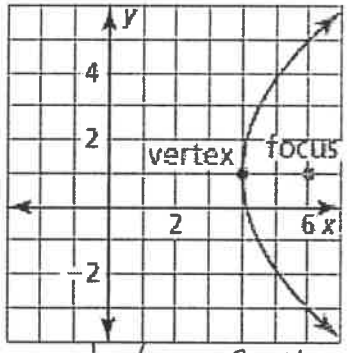
Focus: $(-8, 0)$

$p = -8$

$x = \frac{1}{4(-8)}y^2$

$x = -\frac{1}{32}y^2$

30.



Focus: $(6, 1)$

Vertex: $(4, 1)$

$h = 4 \quad k = 1$

Focus: $(h+p, k)$

$h + p = 6$

$4 + p = 6$

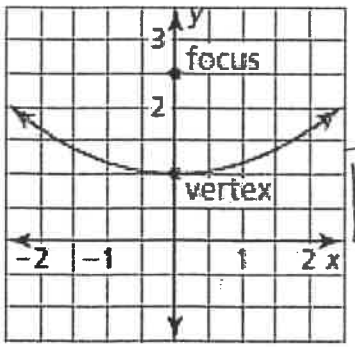
$-4 \quad -4$

$p = 2$

$x = \frac{1}{4(2)}(y-1)^2 + 4$

$x = \frac{1}{8}(y-1)^2 + 4$

31.



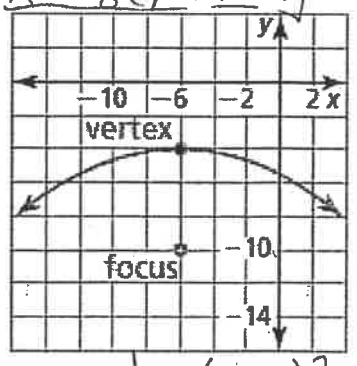
Vertex: $(0, 2)$

$p = 2$

$y = \frac{1}{4(2)}x^2$

$y = \frac{1}{8}x^2$

32.



Vertex: $(-6, -4)$

$h = -6 \quad k = -4$

Focus: $(-6, -10)$

Focus: $(h, k+p)$

$k + p = -10$

$-4 + p = -10$

$+4 \quad +4$

$p = -6$

$y = \frac{1}{4(-6)}(x+6)^2 - 4$

$y = -\frac{1}{24}(x+6)^2 - 4$

College Algebra – Chapter 10
Lesson 3, Day 1

Find the vertices and foci for each ellipse.

1. $\frac{x^2}{16}+\frac{y^2}{4}=1$ $16>4$, so $a^2=16$ $b^2=4$ → Horizontal

$a^2=16$ $b^2=4$
$a=\pm4$ $b=\pm2$
$b^2=a^2-c^2 \rightarrow \frac{4}{-16}=\frac{16-c^2}{-16}$
$\frac{-12}{-1}=\frac{-c^2}{-1} \Rightarrow 12=c^2$ $c=\pm\sqrt{12} \Rightarrow \pm2\sqrt{3}$

Vertices: $(\pm4,0)$
Foci $(\pm2\sqrt{3},0)$

2. $\frac{x^2}{4}+\frac{y^2}{16}=1$ $a^2=16$ $b^2=4$
$a=\pm4$ $b=\pm2$
$b^2=a^2-c^2 \rightarrow \frac{4}{-16}=\frac{16-c^2}{-16}$
$\frac{-12}{-1}=\frac{-c^2}{-1}$
$12=c^2$
$c=\sqrt{12} \Rightarrow \pm2\sqrt{3}$

Vert: $(0,\pm4)$
Foci $(0,\pm2\sqrt{3})$

3. $\frac{x^2}{9}+y^2=1$
$a^2=9$ $b^2=1$
$a=\pm3$ $b=\pm1$
$\frac{1}{-9}=\frac{9-c^2}{-9}$
$\frac{-8}{-1}=\frac{-c^2}{-1}$
$8=c^2$
$c=\pm\sqrt{8}$
$c=\pm2\sqrt{2}$

Vert: $(\pm3,0)$
Foci $(\pm2\sqrt{2},0)$

4. $\frac{x^2}{25}+\frac{y^2}{16}=1$
$a^2=25$ $b^2=16$
$a=\pm5$ $b=\pm4$
$\frac{16}{-25}=\frac{25-c^2}{-25}$
$\frac{-9}{-1}=\frac{-c^2}{-1}$
$9=c^2$
$c=\pm3$

Vert: $(\pm5,0)$
Foci $(\pm3,0)$

5. $x^2+\frac{y^2}{4}=1$
$a^2=4$ $b^2=1$
$a=\pm2$ $b=\pm1$
$\frac{1}{-4}=\frac{4-c^2}{-4}$
$\frac{-3}{-1}=\frac{-c^2}{-1}$
$3=c^2$
$c=\pm\sqrt{3}$

Vert: $(0,\pm2)$
Foci: $(0,\pm\sqrt{3})$

6. $\frac{x^2}{16}+\frac{y^2}{36}=1$
$a^2=36$ $b^2=16$
$a=\pm6$ $c=\pm4$
$16=36-c^2$
$-36\ \ -36$
$-20=-c^2$
$20=c^2$
$c=\pm\sqrt{20}$
$c=\pm2\sqrt{5}$

Vert: $(0,\pm6)$
Foci $(0,\pm2\sqrt{5})$

7. $\frac{x^2}{4}+\frac{y^2}{4}=\frac{4}{4} \Rightarrow \frac{x^2}{4}+\frac{y^2}{4}=1$

$A=B$
this is a circle

8. $\frac{x^2}{16}+\frac{y^2}{16}=\frac{16}{16} \rightarrow \frac{x^2}{16}+\frac{y^2}{16}=1$

$A=B$
this is a circle

9. $\frac{x^2}{4}+\frac{4y^2}{4}=\frac{4}{4} \Rightarrow \frac{x^2}{4}+\frac{y^2}{1}=1$
$a^2=4$ $b^2=1$
$a=\pm2$ $b=\pm1$
$\frac{1}{-4}=\frac{4-c^2}{-4}$
$\frac{-3}{-1}=\frac{-c^2}{-1}$
$3=c^2$
$c=\pm\sqrt{3}$

Vert: $(\pm2,0)$
Foci: $(\pm\sqrt{3},0)$

10. $\frac{9x^2}{9}+\frac{y^2}{9}=\frac{9}{9} \Rightarrow \frac{x^2}{1}+\frac{y^2}{9}=1$
$a^2=9$ $b^2=1$
$a=\pm3$ $b=\pm1$
$\frac{1}{-9}=\frac{9-c^2}{-9}$
$\frac{-8}{-1}=\frac{-c^2}{-1}$
$8=c^2$
$c=\pm\sqrt{8}$
$c=\pm2\sqrt{2}$

Vert: $(0,\pm3)$
Foci: $(0,\pm2\sqrt{2})$

11. $\frac{9x^2}{36}+\frac{4y^2}{36}=\frac{36}{36} \Rightarrow \frac{x^2}{4}+\frac{y^2}{9}=1$
$a^2=9$ $b^2=4$
$a=\pm3$ $b=\pm2$
$\frac{4}{-9}=\frac{9-c^2}{-9}$
$\frac{-5}{-1}=\frac{-c^2}{-1}$
$5=c^2$
$c=\pm\sqrt{5}$

Vert: $(0,\pm3)$
Foci $(0,\pm\sqrt{5})$

12. $\frac{4x^2}{100}+\frac{25y^2}{100}=\frac{100}{100} \Rightarrow \frac{x^2}{25}+\frac{y^2}{4}=1$
$a^2=25$ $b^2=4$
$a=\pm5$ $b=\pm2$
$4=25-c^2$
$-25\ \ -25$
$\frac{-21}{-1}=\frac{-c^2}{-1}$
$21=c^2$
$c=\pm\sqrt{21}$

Vert: $(\pm5,0)$
Foci: $(\pm\sqrt{21},0)$

College Algebra – Chapter 10
Lesson 3, Day 1

Find the Standard form of the equation for the ellipse satisfying the given conditions.

13. Foci: (±1, 0); vertex (3,0) Major Axis: x-axis Horizontal

$a=3 \quad c=1$

$b^2=(3)^2-(1)^2$

$b^2=9-1$

$b^2=8$

$$\frac{x^2}{9}+\frac{y^2}{8}=1$$

14. Foci: (±3, 0); vertex (5,0) Horizontal

$c=3 \quad a=5$

$b^2=(5)^2-(3)^2$

$b^2=25-9$

$b^2=16$

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

15. Foci: (0, ±2); vertex (0,4) Vertical

$a=4 \quad c=2$

$b^2=(4)^2-(2)^2$

$b^2=16-4$

$b^2=12$

$$\frac{x^2}{12}+\frac{y^2}{16}=1$$

16. Foci: (0, ±3); vertex (0, -6) Vertical

$a=-6 \quad c=3$

$b^2=(-6)^2-(3)^2$

$b^2=36-9$

$b^2=25$

$$\frac{x^2}{25}+\frac{y^2}{36}=1$$

17. Foci: (±4, 0); y-intercepts: ±3 Horizontal

$c=4 \quad b=3$

$(3)^2=a^2-(4)^2$

$9=a^2-16$

$+16 \quad +16$

$25=a^2$

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

18. Foci: (±3, 0); y-intercepts: ±4 Horizontal

$c=3 \quad b=4$

$(4)^2=a^2-(3)^2$

$16=a^2-9$

$+9 \quad +9$

$25=a^2$

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

19. Foci: (0, ±2); x-intercepts: ±4 Vertical

$b=4 \quad c=2$

$(4)^2=a^2-(2)^2$

$16=a^2-4$

$+4 \quad +4$

$20=a^2$

$$\frac{x^2}{16}+\frac{y^2}{20}=1$$

20. Foci: (0, ±3); x-intercepts: ±5 Vertical

$b=5 \quad c=3$

$5^2=a^2-(3)^2$

$25=a^2-9$

$+9 \quad +9$

$36=a^2$

$$\frac{x^2}{25}+\frac{y^2}{36}=1$$

College Algebra – Chapter 10
Lesson 3, Day 2

Find the center, foci, and vertices of each ellipse.

1. $\frac{(x-1)^2}{4} + \frac{(y-1)^2}{9} = 1$ Vertical

Center $(1, 1)$

$a^2 = 9 \quad b^2 = 4$
$a = \pm 3 \quad b = \pm 2$
$c^2 = 9 - 4$
$c^2 = 5$
$c = \pm\sqrt{5}$

Foci: $(h, k \pm c)$
$(1, 1 \pm \sqrt{5})$

Vert: $(h, k \pm a)$
$(1, 1 \pm 3)$
$(1, -2) + (1, 4)$

2. $\frac{(x-1)^2}{4} + \frac{(y-1)^2}{16} = 1$ Vertical

$a^2 = 16 \quad b^2 = 4$
$a = \pm 4 \quad b = \pm 2$
$c^2 = 16 - 4$
$c^2 = 12$
$c = \pm\sqrt{12}$
$c = \pm 2\sqrt{3}$

Center: $(1, 1)$

Foci: $(h, k \pm c)$
$(1, 1 \pm 2\sqrt{3})$

Vert: $(h, k \pm a)$
$(1, 1 \pm 4)$
$(1, -3) + (1, 5)$

3. $\frac{x^2}{16} + \frac{(y+3)^2}{4} = 1$ Horizontal

$a^2 = 16 \quad b^2 = 4$
$a = \pm 4 \quad b = \pm 2$
$c^2 = 16 - 4$
$c^2 = 12$
$c = \pm\sqrt{12}$
$c = \pm 2\sqrt{3}$

Center: $(0, -3)$

Foci: $(h \pm c, k)$
$(0 \pm 2\sqrt{3}, -3)$

Vert: $(h \pm a, k)$
$(0 \pm 4, -3)$
$(4, -3) + (-4, -3)$

4. $\frac{(x+2)^2}{4} + \frac{y^2}{9} = 1$ Vertical

$a^2 = 9 \quad b^2 = 4$
$a = \pm 3 \quad b = \pm 2$
$c^2 = 9 - 4$
$c^2 = 5$
$c = \pm\sqrt{5}$

Center: $(-2, 0)$

Foci: $(h, k \pm c)$
$(-2, 0 \pm \sqrt{5})$

Vert: $(h, k \pm a)$
$(-2, 0 \pm 3)$
$(-2, 3) + (-2, -3)$

5. $\frac{3(x-1)^2}{12} + \frac{4(y+2)^2}{12} = \frac{12}{12}$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{3} = 1$ Horizontal

$a^2 = 4 \quad b^2 = 3$
$a = \pm 2 \quad b = \pm\sqrt{3}$
$c^2 = 4 - 3$
$c^2 = 1$
$c = \pm 1$

Center: $(1, -2)$

Foci: $(h \pm c, k)$
$(1 \pm 1, -2)$
$(2, -2) + (0, -2)$

Vert: $(h \pm a, k)$
$(1 \pm 2, -2)$
$(3, -2) + (-1, -2)$

6. $\frac{9(x+1)^2}{36} + \frac{4(y-2)^2}{36} = \frac{36}{36}$

$\frac{(x+1)^2}{4} + \frac{(y-2)^2}{9} = 1$ Vertical

$a^2 = 9 \quad b^2 = 4$
$a = \pm 3 \quad b = \pm 2$
$c^2 = 9 - 4$
$c^2 = 5$
$c = \pm\sqrt{5}$

Center: $(-1, 2)$

Foci: $(h, k \pm c)$
$(-1, 2 \pm \sqrt{5})$

Vert: $(h, k \pm a)$
$(-1, 2 \pm 3)$
$(-1, 5) + (-1, -1)$

$C=\left(\frac{b}{2}\right)^2$

College Algebra – Chapter 10
Lesson 3, Day 2

7. $5x^2 + 9y^2 + 10x - 36y - 4 = 0$

$5x^2+10x+9y^2-36y=4$

$5(x^2+2x)+9(y^2-4y)=4$

$C=\left(\frac{2}{2}\right)^2=1^2=1 \quad C=\left(\frac{-4}{2}\right)^2=(-2)^2=4$

$5(x^2+2x+1)+9(y^2-4y+4)=4+5+36$

5 · 1 = 5; 9 · 4 = 36

$\frac{5(x+1)^2}{45}+\frac{9(y-2)^2}{45}=\frac{45}{45}$

$\frac{(x+1)^2}{9}+\frac{(y-2)^2}{5}=1$ Horizontal

$a^2=9 \quad b^2=5$

$a=\pm3 \quad b=\pm\sqrt{5}$

$c^2=9-5$

$c^2=4$

$c=\pm2$

Center: $(-1,2)$
Foci: $(h\pm c, k)$
$(-1\pm2, 2)$
$(1,2)+(-3,2)$
Vert: $(h\pm a, k)$
$(-1\pm3, 2)$
$(2,2)+(-4,2)$

8. $9x^2 + 4y^2 + 72x - 24y + 144 = 0$

$9x^2+72x+4y^2-24y=-144$

$9(x^2+8x)+4(y^2-6y)=-144$

$C=\left(\frac{8}{2}\right)^2=4^2=16 \quad C=\left(\frac{-6}{2}\right)^2=-3^2=9$

$9(x^2+8x+16)+4(y^2-6y+9)=-144+144+36$

9 · 16 = 144; 4 · 9 = 36

$\frac{9(x+4)^2}{36}+\frac{4(y-3)^2}{36}=\frac{36}{36}$

$\frac{(x+4)^2}{4}+\frac{(y-3)^2}{9}=1$ Vertical

$a^2=9 \quad b^2=4$

$a=\pm3 \quad b=\pm2$

$c^2=9-4$

$c^2=5$

$c=\pm\sqrt{5}$

Center: $(-4,3)$
Foci: $(h, k\pm c)$
$(-4, 3\pm\sqrt{5})$
Vert: $(h, k\pm a)$
$(-4, 3\pm3)$
$(-4,6)+(-4,0)$

9. $9x^2 + 5y^2 + 36x - 40y + 71 = 0$

$9x^2+36x+5y^2-40y=-71$

$9(x^2+4x)+5(y^2-8y)=-71$

$C=\left(\frac{4}{2}\right)^2=2^2=4 \quad C=\left(\frac{-8}{2}\right)^2=-4^2=16$

$9(x^2+4x+4)+5(y^2-8y+16)=-71+36+80$

9 · 4 = 36; 5 · 16 = 80

$\frac{9(x+2)^2}{45}+\frac{5(y-4)^2}{45}=\frac{45}{45}$

$\frac{(x+2)^2}{5}+\frac{(y-4)^2}{9}=1$ Vertical

$a^2=9 \quad b^2=5$

$a=\pm3 \quad b=\pm\sqrt{5}$

$c^2=9-5$

$c^2=4$

$c=\pm2$

Center: $(-2,4)$
Foci: $(h, k\pm c)$
$(-2, 4\pm2)$
$(-2,6)+(-2,2)$
Vert: $(h, k\pm a)$
$(-2, 4\pm3)$
$(-2,7)+(-2,1)$

10. $3x^2 + 4y^2 + 12x - 16y - 32 = 0$

$3x^2-12x+4y^2-16y=32$

$3(x^2-4x)+4(y^2-4y)=32$

$C=\left(\frac{-4}{2}\right)^2=-2^2=4 \quad C=\left(\frac{-4}{2}\right)^2=(-2)^2=4$

$3(x^2-4x+4)+4(y^2-4y+4)=32+12+16$

3 · 4 = 12; 4 · 4 = 16

$\frac{3(x-2)^2}{60}+\frac{4(y-2)^2}{60}=\frac{60}{60}$

$\frac{(x-2)^2}{20}+\frac{(y-2)^2}{15}=1$ Horizontal

$a^2=20 \quad b^2=15$

$a=\pm\sqrt{20} \quad b=\pm\sqrt{15}$

$a=\pm2\sqrt{5}$

$c^2=20-15$

$c^2=5$

$c=\pm\sqrt{5}$

Center: $(2,2)$
Foci: $(h\pm c, k)$
$(2\pm\sqrt{5}, 2)$
Vert: $(h\pm a, k)$
$(2\pm2\sqrt{5}, 2)$

College Algebra – Chapter 10
Lesson 4, Day 1

Find the vertices, foci, and transverse axis of the hyperbola.

1. $x^2 - \frac{y^2}{4} = 1$. Horizontal Tranverse

$a^2=1 \quad b^2=4$
$a=\pm1 \quad b=\pm2$
$c^2=1+4$
$c^2=5$
$c=\pm\sqrt{5}$

Vert: $(\pm1, 0)$
Foci: $(\pm\sqrt{5}, 0)$
Trans Axis: X-axis
Length: $2(1)=2$

2. $y^2 - \frac{x^2}{4} = 1$ Vertical Transverse

$a^2=1 \quad b^2=4$
$a=\pm1 \quad b=\pm2$
$c^2=1+4$
$c^2=5$
$c=\pm\sqrt{5}$

Vert: $(0, \pm1)$
Foci $(0, \pm\sqrt{5})$
Trans Axis: y-axis
Length: $2(1)=2$

3. $x^2 - y^2 = -1$
$\frac{}{-1} \quad \frac{}{-1} \quad \frac{}{-1}$
$-x^2+y^2=1 \Rightarrow y^2-x^2=1$ Horizontal Transverse

$a^2=1 \quad b^2=1$
$a=\pm1 \quad b=\pm1$
$c^2=1+1$
$c^2=2$
$c=\pm\sqrt{2}$

Vert: $(\pm1, 0)$
Foci $(\pm\sqrt{2}, 0)$
Trans Axis: X-axis
length: $2(1)=2$

4. $\frac{9y^2}{36} - \frac{x^2}{36} = \frac{36}{36}$ Vertical Transverse

$\frac{y^2}{4} - \frac{x^2}{36} = 1$
$a^2=4 \quad b^2=36$
$a=\pm2 \quad b=\pm6$
$c^2=4+36$
$c^2=40$
$c=\pm\sqrt{40}$
$c=\pm2\sqrt{10}$

Vert: $(0, \pm2)$
Foci: $(0, \pm2\sqrt{10})$
Trans Axis: y-axis
length: $2(2)=4$

5. $4x^2 - 9y^2 + 36 = 0$
$\frac{4x^2}{-36} - \frac{9y^2}{-36} = \frac{-36}{-36}$
$\frac{-x^2}{9} + \frac{y^2}{4} = 1 \Rightarrow \frac{y^2}{4} - \frac{x^2}{9} = 1$ Vertical Transverse

$a^2=4 \quad b^2=9$
$a=\pm2 \quad b=\pm3$
$c^2=4+9$
$c^2=13$
$c=\pm\sqrt{13}$

Vert: $(0, \pm2)$
Foci $(0, \pm\sqrt{13})$
Trans Axis: y-axis
Length: $2(2)=4$

6. $(y)^2=(\pm\sqrt{4x^2+1})^2$ Vertical Transverse
$y^2=4x^2+1$
$-4x^2 \quad -4x^2$
$y^2-4x^2=1 \Rightarrow \frac{y^2}{1} - \frac{x^2}{\frac{1}{4}} = 1$

$a^2=1 \quad b^2=\frac{1}{4}$
$a=\pm1 \quad b=\pm\frac{1}{2}$
$c^2=1+\frac{1}{4}$
$c^2=\frac{4}{4}+\frac{1}{4}$
$c^2=\frac{5}{4}$
$c=\pm\frac{\sqrt{5}}{\sqrt{4}}$
$c=\pm\frac{\sqrt{5}}{2}$

Vert: $(0, \pm1)$
Foci: $(0, \pm\frac{\sqrt{5}}{2})$
Trans Axis: y-axis
Length $2(1)=2$

College Algebra – Chapter 10
Lesson 4, Day 1

Find an equation of the hyperbola satisfying the given conditions.

7. Vertices: (±2, 0); Foci: (±3, 0)

$a = \pm 2 \qquad c = \pm 3$
$a^2 = 4 \qquad c^2 = 9$
$9 = 4 + b^2$
$-4 \quad -4$
$5 = b^2$

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

8. Vertices: (±3, 0); Foci: (±5, 0)

$a = \pm 3 \qquad c = \pm 5$
$a^2 = 9 \qquad c^2 = 25$
$25 = 9 + b^2$
$-9 \quad -9$
$16 = b^2$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

8. Center (0, 0) Vertex (0, 2) Focus (0, 5)

$a = 2 \qquad c = 5$
$a^2 = 4 \qquad c^2 = 25$
$25 = 4 + b^2$
$-4 \quad -4$
$21 = b^2$

$$\frac{y^2}{4} - \frac{x^2}{21} = 1$$

9. Center (0, 0); Vertex: (0, -1); Focus (0, -4)

$a = -1 \qquad c = -4$
$a^2 = 1 \qquad c^2 = 16$
$16 = 1 + b^2$
$-1 \quad -1$
$15 = b^2$

$$\frac{y^2}{1} - \frac{x^2}{15} = 1$$

10. Foci: (0, ±5); Length of transverse axis: 6

$c = 5 \qquad \frac{2a}{2} = \frac{6}{2}$
$c^2 = 25 \qquad a = 3$
$a^2 = 9$

$25 = 9 + b^2$
$-9 \quad -9$
$16 = b^2$

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

11. Foci: (±2, 0); Length of transverse axis: 2

$c = \pm 2 \qquad \frac{2a}{2} = \frac{2}{2}$
$c^2 = 4 \qquad a = 1$
$a^2 = 1$

$4 = 1 + b^2$
$-1 \quad -1$
$3 = b^2$

$$\frac{x^2}{1} - \frac{y^2}{3} = 1$$

College Algebra – Chapter 10
Lesson 4, Day 2

Graph the hyperbola.

1. $\frac{x^2}{2} - \frac{y^2}{4} = 1$. $a^2=2 \to a=\sqrt{2}$, $b^2=4 \to b=\pm 2$

Transverse $\longrightarrow$ x-axis $\to$ opens L+R

Asymp: $y = \frac{b}{a}x \Rightarrow \frac{2}{\sqrt{2}}x \Rightarrow \frac{2\sqrt{2}}{2}x \Rightarrow \sqrt{2}x$

$y = -\frac{b}{a}x \Rightarrow -\frac{2}{\sqrt{2}}x = -\frac{2\sqrt{2}}{2}x \Rightarrow -\sqrt{2}x$

Vert: $(\pm a, 0) \to (\pm\sqrt{2}, 0)$

Endpoints of Conj Axis: $(0, \pm b) \to (0, \pm 2)$

Fund. Rect: $(\sqrt{2}, 2), (-\sqrt{2}, 2), (\sqrt{2}, -2), (-\sqrt{2}, -2)$

$*\sqrt{2} \approx 1.4$

2. $\frac{x^2}{6} - \frac{y^2}{2} = 1$ $a^2=6$ $a=\pm\sqrt{6}$, $b^2=2$ $b=\pm\sqrt{2}$

Transverse $\to$ x-axis $\to$ opens L+R

Vert: $(\pm a, 0) \to (\pm\sqrt{6}, 0)$

Endpoints of Conj Axis $\to (0, \pm b) \to (0, \pm\sqrt{2})$

Fund. Rect: $(\sqrt{6}, \sqrt{2}), (-\sqrt{6}, \sqrt{2}), (\sqrt{6}, -\sqrt{2})$
$(-\sqrt{6}, -\sqrt{2})$

$*\sqrt{6} \approx 2.4$
$*\sqrt{2} \approx 1.4$

3. $\frac{2y^2}{6} - \frac{x^2}{6} = \frac{6}{6}$ $a^2=3$ $a=\pm\sqrt{3}$, $b^2=16$ $b=\pm\sqrt{16}$

$\frac{y^2}{3} - \frac{x^2}{16} = 1$

Transverse $\to$ y-axis $\to$ opens U+D

Vert: $(0, \pm a) \to (0, \pm\sqrt{3})$

Eq CA: $(\pm b, 0) \to (\pm\sqrt{16}, 0)$

Fund. Rect: $(\sqrt{16}, \sqrt{3}), (\sqrt{16}, -\sqrt{3}),$
$(-\sqrt{16}, \sqrt{3}), (-\sqrt{16}, -\sqrt{3})$

$*\sqrt{3} \approx 1.7$

$*\sqrt{16} \approx .4$

4. $\frac{y^2}{12} - \frac{3x^2}{12} = \frac{12}{12}$ $a^2=12$ $a=\pm\sqrt{12}$

$\frac{y^2}{12} - \frac{x^2}{4} = 1$ $b^2=4$ $b=\pm 2$

Transverse Axis $\to$ y-axis $\to$ opens U+D

Vert: $(0, \pm a) \to (0, \pm\sqrt{12})$

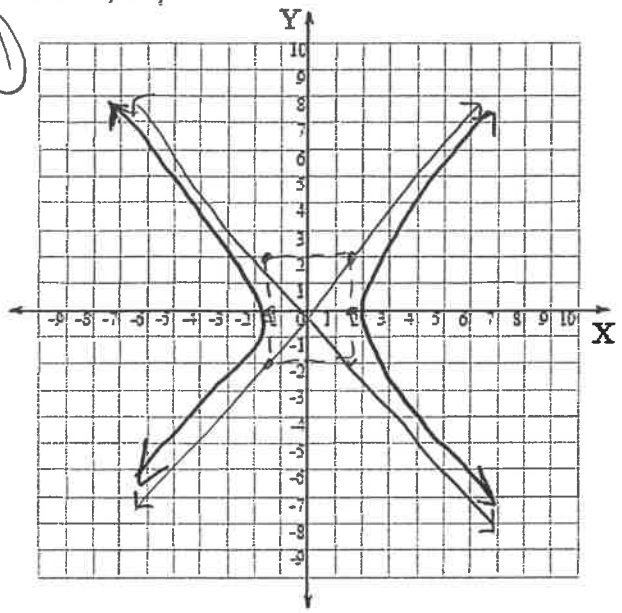
Eq CA: $(\pm b, 0) \to (\pm 2, 0)$

Fund. Rect: $(2, \sqrt{12}), (2, -\sqrt{12}),$
$(-2, \sqrt{12}), (-2, -\sqrt{12})$

$*\sqrt{12} \approx 3.5$

College Algebra – Chapter 10
Lesson 4, Day 2

1



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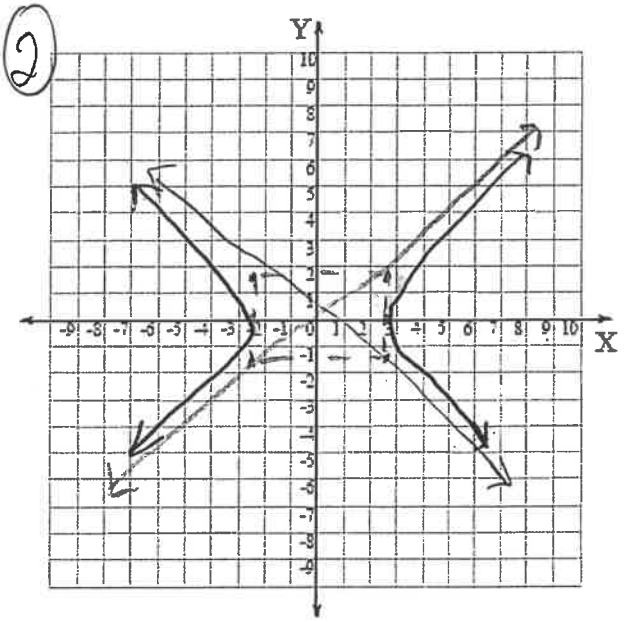

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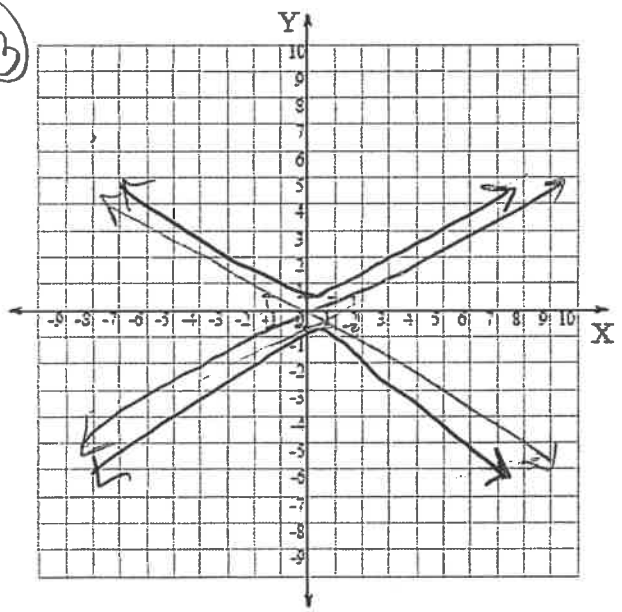

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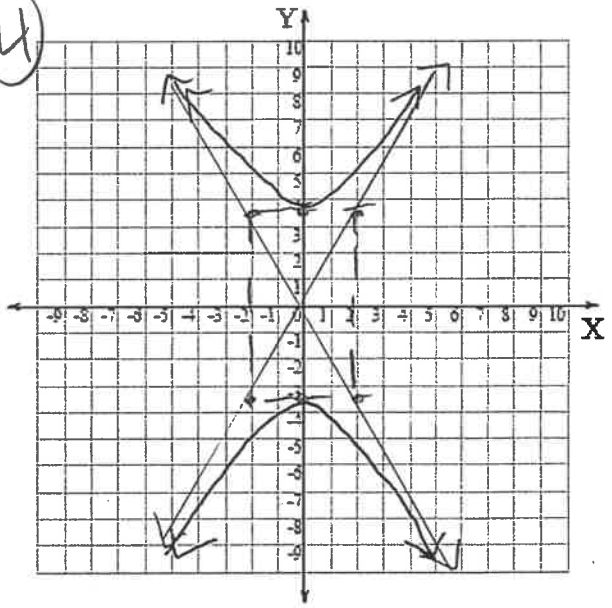
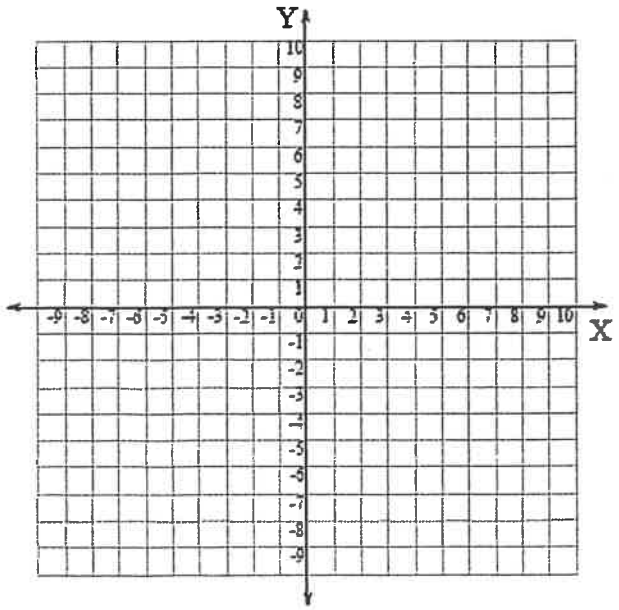
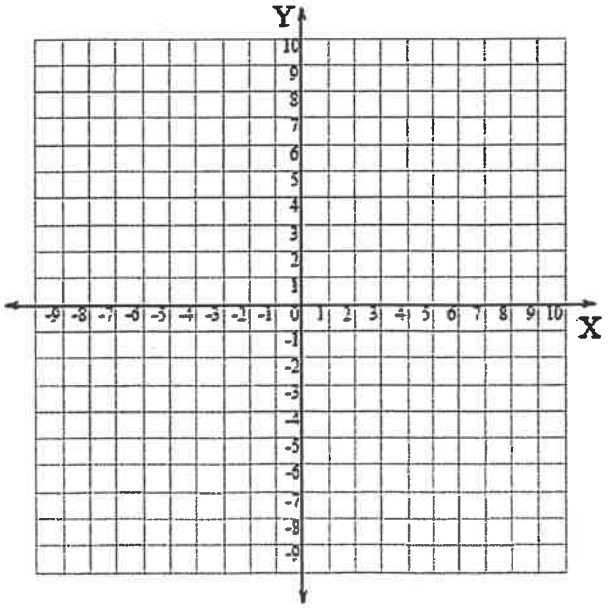






College Algebra – Chapter 10
Lesson 4, Day 3

Endpoints of Conjugate Axis.

Find the Center, Vertices, Transverse Axis, and [illegible]. Then, graph the hyperbola.

1. $\frac{(x-1)^2}{9} - \frac{(y+1)^2}{16} = 1$

$a^2 = 9 \quad a = \pm 3$
$b^2 = 16 \quad b = \pm 4$
Center: $(1, -1)$
Trans. Axis $y = -1$
Vert: $(h \pm a, k) \to (1 \pm 3, -1)$
$(4, -1) + (-2, -1)$
Eff CA: $(h, k \pm b) \to (1, -1 \pm 4)$
$(1, -5) + (1, 3)$

opens L + R

2. $\frac{(y-1)^2}{16} - \frac{(x+1)^2}{4} = 1$

$a^2 = 16 \quad a = \pm 4$
$b^2 = 4 \quad b = \pm 2$
Center: $(-1, 1)$
Trans. Axis: $x = -1$
Vert: $(h, k \pm a) \to (-1, 1 \pm 4)$
$(-1, 5) + (-1, -3)$
Eff CA: $(h \pm b, k) \to (-1 \pm 2, 1)$
$(-3, 1) + (1, 1)$

opens U + D

3. $\frac{(x+2)^2}{25} - \frac{y^2}{49} = 1$

$a^2 = 25 \quad a = \pm 5$
$b^2 = 49 \quad b = \pm 7$
Center: $(-2, 0)$
Trans Axis $y = 0$
Vert: $(h \pm a, k) \to (-2 \pm 5, 0)$
$(-7, 0) + (3, 0)$
Eff CA: $(h, k \pm b) \to (-2, 0 \pm 7)$
$(-2, 7) + (-2, -7)$

opens L + R

4. $\frac{(x+1)^2}{9} - \frac{(y+2)^2}{36} = 1$

$a^2 = 9 \quad a = \pm 3$
$b^2 = 36 \quad b = \pm 6$
Center: $(-1, -2)$
Trans. Axis $\to y = -2$
Vert: $(h \pm a, k) \to (-1 \pm 3, -2)$
$(-4, -2) + (2, -2)$
Eff CA: $(h, k \pm b) \to (-1, -2 \pm 6)$
$(-1, -8) + (-1, 4)$

opens L + R

5. $\frac{4x^2}{25} - \frac{(y+1)^2}{25} = \frac{25}{25}$

$\frac{x^2}{\frac{25}{4}} - \frac{(y+1)^2}{25} = 1$

$a^2 = \frac{25}{4} \quad a = \pm \frac{5}{2}$
$b^2 = 25 \quad b = \pm 5$
Center: $(0, -1)$
Trans. Axis $\to y = -1$
Vert: $(h \pm a, k) \to (0 \pm \frac{5}{2}, -1)$
$(\frac{5}{2}, -1) + (-\frac{5}{2}, -1)$
Eff CA: $(h, k \pm b) \Rightarrow (0, -1 \pm 5)$
$(0, -6) + (0, 4)$

opens L + R

6. $\frac{9(x-1)^2}{144} - \frac{y^2}{144} = \frac{144}{144}$

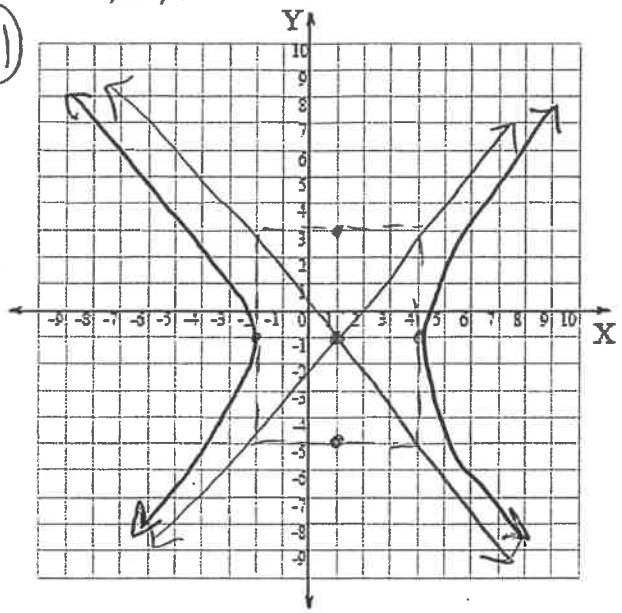
$\frac{(x-1)^2}{16} - \frac{y^2}{12} = 1$

$a^2 = 16 \quad a = \pm 4$
$b^2 = 144 \quad b = \pm 12$
Center: $(1, 0)$
Trans Axis $y = 0$
Vert: $(h \pm a, k) \to (1 \pm 4, 0)$
$(-3, 0) + (5, 0)$
Eff CA: $(h, k \pm b) \Rightarrow (1, 0 \pm 12)$
$(1, 12) + (1, -12)$

opens L + R

College Algebra – Chapter 10
Lesson 4, Day 3

1



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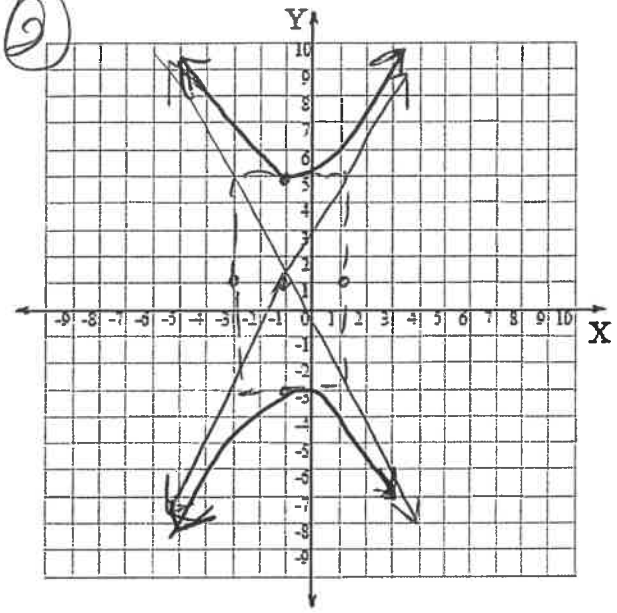

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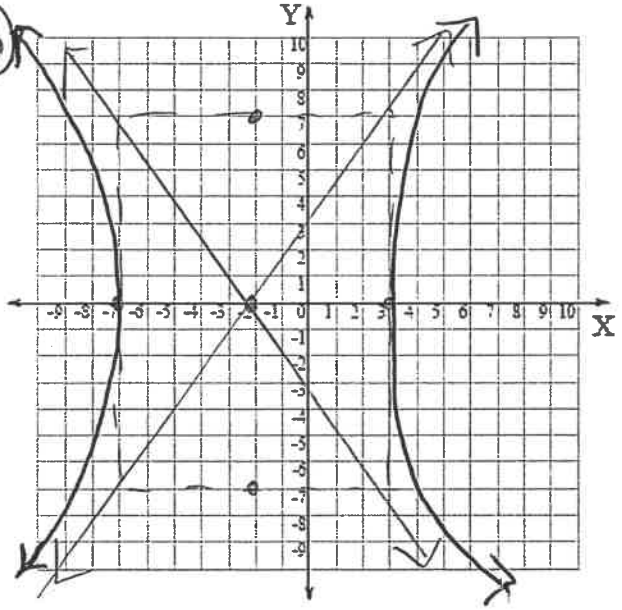

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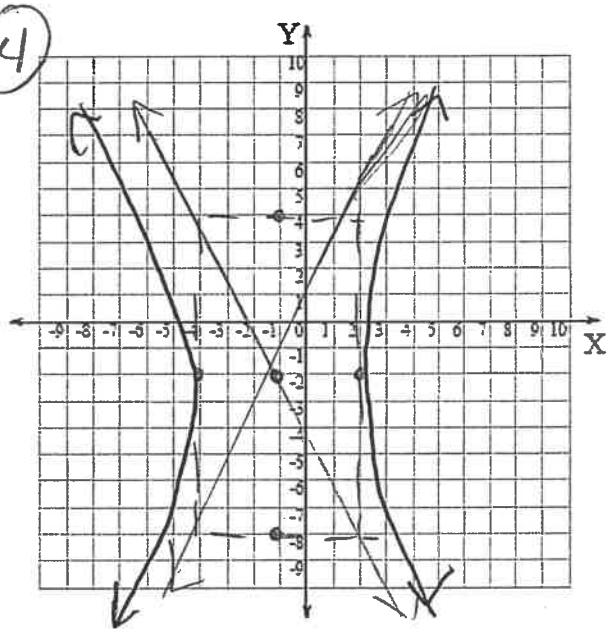

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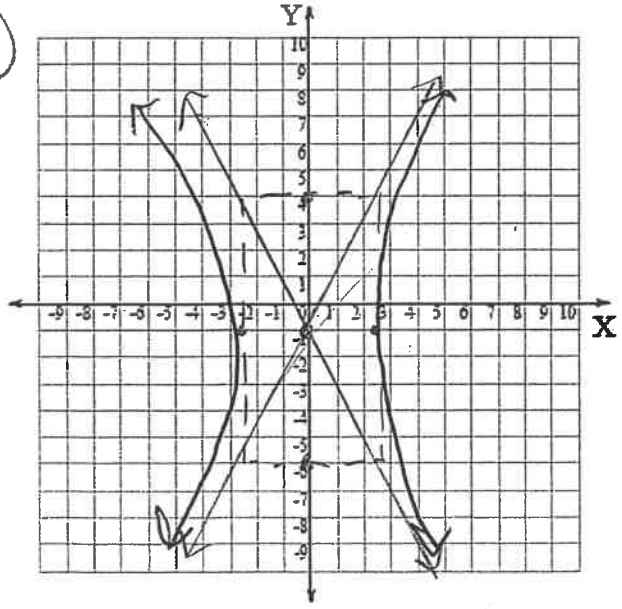
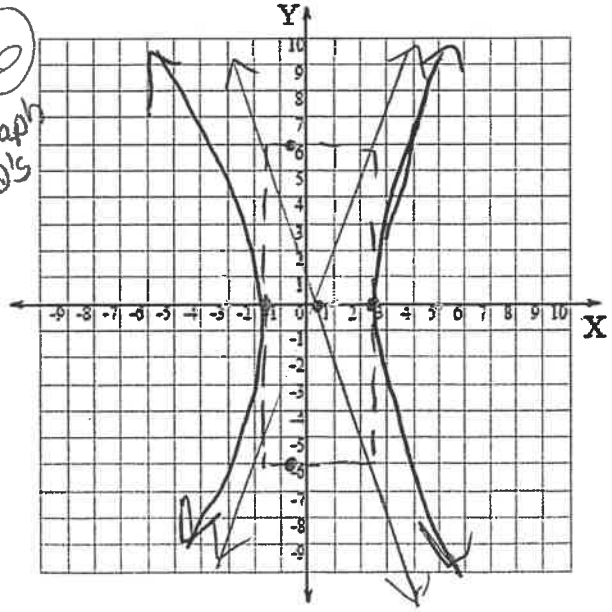


6

*graph by 2's



College Algebra – Chapter 10
Lesson 4, Day 3

7. $\frac{(y+1)^2}{25} - \frac{9(x-2)^2}{25} = \frac{25}{25}$

$\frac{(y+1)^2}{25} - \frac{(x-2)^2}{25/9} = 1$

$a^2 = 25 \quad a = \pm 5$

$b^2 = \frac{25}{9} \quad b = \pm\frac{5}{3}$

Center: $(2, -1)$

Trans Axis: $x = 2$

Vert: $(h, k \pm a) \rightarrow (2, -1 \pm 5)$

$(2, -6)$ & $(2, 4)$

Ef CA: $(h \pm b, k) \rightarrow (2 \pm \frac{5}{3}, -1)$

$(\frac{11}{3}, -1)$ & $(\frac{1}{3}, -1)$

opens U+D

8. $\frac{6(x-4)^2}{4} - \frac{3(y+3)^2}{4} = \frac{4}{4}$

$\frac{(x-4)^2}{\frac{4}{3}} - \frac{(y+3)^2}{\frac{4}{3}} = 1$

$a^2 = \frac{2}{3} \quad a = \pm\sqrt{3/2} \approx 1.2$

$b^2 = \frac{4}{3} \quad b = \pm\sqrt{4/3} \approx 1.2$

Center: $(4, -3)$

Trans. Axis $\rightarrow y = -3$

Vert: $(h \pm a, k) \rightarrow (4 \pm 1.2, -3)$

$(5.2, -3)$ & $(2.8, -3)$

Ef CA: $(h, k \pm b) \rightarrow (4, -3 \pm 1.2)$

$(4, -4.2)$ & $(4, -1.8)$

opens L+R

9. $x^2 - 4y^2 - 4 = 0$

$\frac{x^2}{4} - \frac{4y^2}{4} = \frac{4}{4}$

$\frac{x^2}{4} - \frac{y^2}{2} = 1$

$a^2 = 4 \quad a = \pm 2$

$b^2 = 2 \quad b = \pm\sqrt{2} \approx 1.4$

Center: $(0, 0)$

Trans. Axis $\rightarrow y = 0$

Vert: $(h \pm a, k) \rightarrow (0 \pm 2, 0)$

$(-2, 0)$ & $(2, 0)$

Ef CA: $(h, k \pm b) \rightarrow (0, 0 \pm 1.4)$

$(0, 1.4)$ & $(0, -1.4)$

opens L+R

10. $2x^2 - y^2 + 12x - 8y + 3 = 0$

$2x^2 + 12x - y^2 - 8y = -3$

$2(x^2 + 6x) - (y^2 + 8y) = -3$

$c = (\frac{6}{2})^2 = 3^2 = 9 \qquad c = (\frac{8}{2})^2 = 4^2 = 16$

$2(x^2 + 6x + 9) - (y^2 + 8y + 16) = -3 + 18 - 16$

$18 \qquad -16$

$\frac{2(x+3)^2}{-1} - \frac{(y+4)^2}{-1} = \frac{-1}{-1}$

$-\frac{(x+3)^2}{1/2} + (y+4)^2 = 1$

$(y+4)^2 - \frac{(x+3)^2}{1/2} = 1$

$a^2 = 1 \quad a = \pm 1$

$b^2 = 1/2 \quad b = \pm\sqrt{1/2} \approx .7$

Center: $(-3, -4)$

Trans Axis: $x = -3$

Vert: $(h, k \pm a) \rightarrow (-3, -4 \pm 1)$

$(-3, -5)$ & $(-3, -3)$

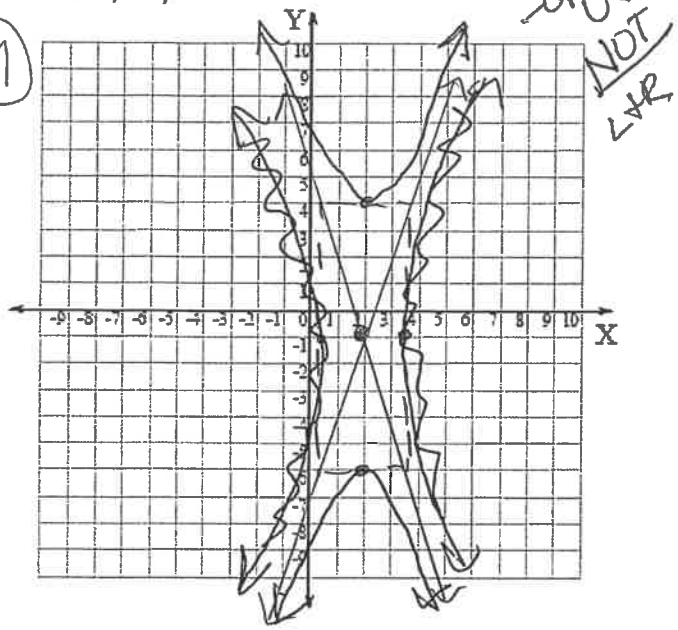
Ef CA: $(h \pm b, k) \rightarrow (-3 \pm .7, -4)$

$(-3.7, -4)$ & $(-2.3, -4)$

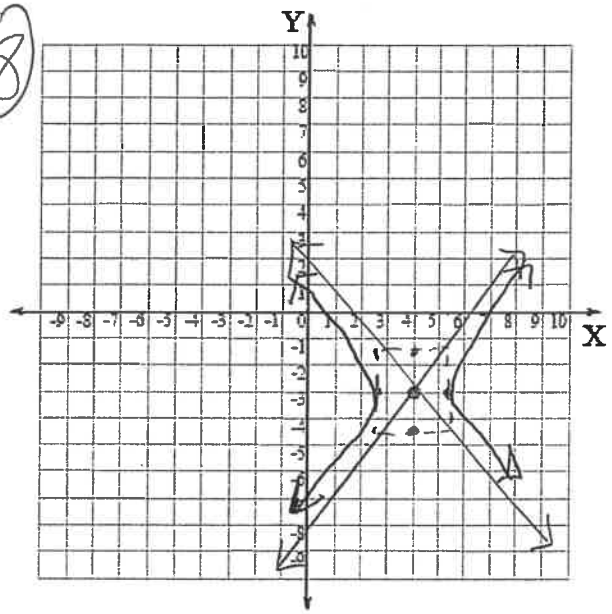
opens U+D

College Algebra – Chapter 10
Lesson 4, Day 3

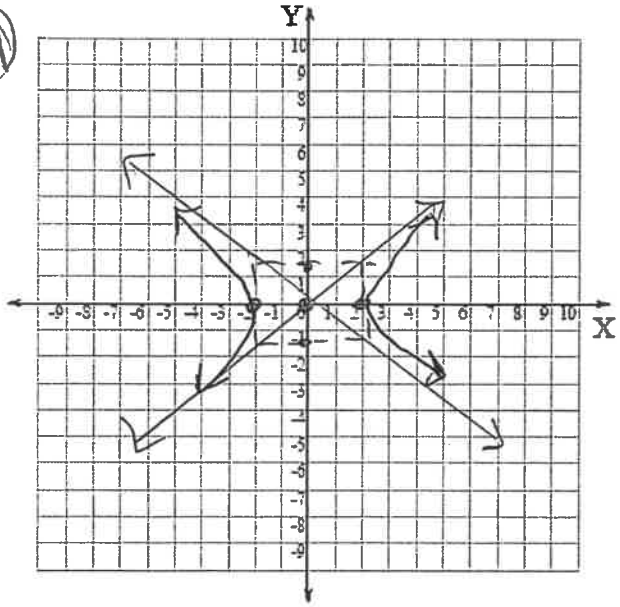
7



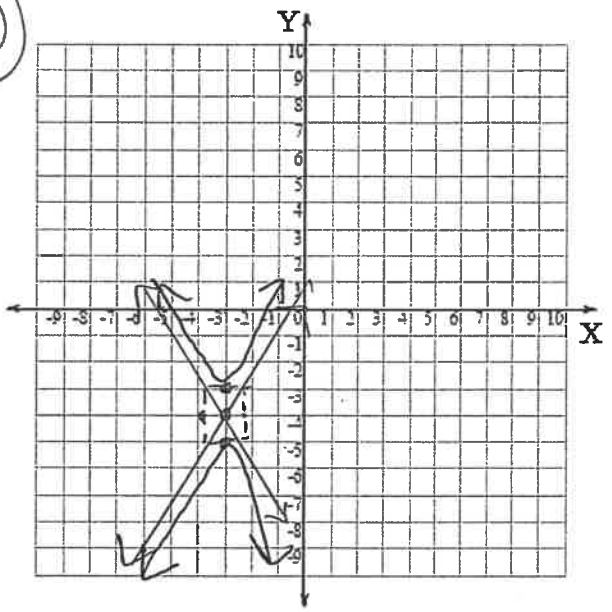
8



9



10



College Algebra – Chapter 11
Lesson 1, Day 1

Write the first six terms of the sequence.

1. $a_n = n + 2$

$a_1 \Rightarrow 1+2 = 3$
$a_2 \Rightarrow 2+2 = 4$
$a_3 \Rightarrow 3+2 = 5$
$a_4 \Rightarrow 4+2 = 6$
$a_5 \Rightarrow 5+2 = 7$
$a_6 \Rightarrow 6+2 = 8$

2. $a_n = 6 - n$

$a_1 \Rightarrow 6-1 = 5$
$a_2 \Rightarrow 6-2 = 4$
$a_3 \Rightarrow 6-3 = 3$
$a_4 \Rightarrow 6-4 = 2$
$a_5 \Rightarrow 6-5 = 1$
$a_6 \Rightarrow 6-6 = 0$

3. $a_n = n^2$

$a_1 = (1)^2 = 1$
$a_2 \Rightarrow 2^2 = 4$
$a_3 \Rightarrow 3^2 = 9$
$a_4 \Rightarrow 4^2 = 16$
$a_5 \Rightarrow 5^2 = 25$
$a_6 \Rightarrow 6^2 = 36$

4. $f(n) = n^3 + 2$

$a_1 = 1^3+2 \Rightarrow 1+2 = 3$
$a_2 = 2^3+2 \Rightarrow 8+2 = 10$
$a_3 = 3^3+2 \Rightarrow 27+2 = 29$
$a_4 = 4^3+2 \Rightarrow 64+2 = 66$
$a_5 = 5^3+2 \Rightarrow 125+2 = 127$
$a_6 = 6^3+2 \Rightarrow 216+2 = 218$

5. $f(n) = 4^{n-1}$

$a_1 = 4^{1-1} \Rightarrow 4^0 = 1$
$a_2 = 4^{2-1} \Rightarrow 4^1 = 4$
$a_3 = 4^{3-1} \Rightarrow 4^2 = 16$
$a_4 = 4^{4-1} \Rightarrow 4^3 = 64$
$a_5 = 4^{5-1} \Rightarrow 4^4 = 256$
$a_6 = 4^{6-1} \Rightarrow 4^5 = 1024$

6. $a_n = -n^2$

$a_1 = -(1)^2 = -1$
$a_2 = -(2)^2 = -4$
$a_3 = -(3)^2 = -9$
$a_4 = -(4)^2 = -16$
$a_5 = -(5)^2 = -25$
$a_6 = -(6)^2 = -36$

7. $a_n = n^2 - 5$

$a_1 = 1^2 - 5 \Rightarrow 1-5 = -4$
$a_2 = 2^2 - 5 \Rightarrow 4-5 = -1$
$a_3 = 3^2 - 5 \Rightarrow 9-5 = 4$
$a_4 = 4^2 - 5 \Rightarrow 16-5 = 11$
$a_5 = 5^2 - 5 \Rightarrow 25-5 = 20$
$a_6 = 6^2 - 5 \Rightarrow 36-5 = 29$

8. $a_n = (n+3)^2$

$a_1 = (1+3)^2 \Rightarrow (4)^2 = 16$
$a_2 = (2+3)^2 \Rightarrow 5^2 = 25$
$a_3 = (3+3)^2 \Rightarrow 6^2 = 36$
$a_4 = (4+3)^2 \Rightarrow 7^2 = 49$
$a_5 = (5+3)^2 \Rightarrow 8^2 = 64$
$a_6 = (6+3)^2 \Rightarrow 9^2 = 81$

9 $f(n) = \frac{2n}{n+2}$.

$F(1) = \frac{2(1)}{1+2} = \frac{2}{3}$
$F(2) = \frac{2(2)}{2+2} = \frac{4}{4} = 1$
$F(3) = \frac{2(3)}{3+2} = \frac{6}{5}$
$F(4) = \frac{2(4)}{4+2} = \frac{2}{6} = \frac{4}{3}$
$F(5) = \frac{2(5)}{5+2} = \frac{10}{7}$
$F(6) = \frac{2(6)}{6+2} = \frac{12}{8} = \frac{3}{2}$

10. $f(n) = \frac{n}{2n-1}$

$f(1) = \frac{1}{2(1)-1} \Rightarrow \frac{1}{2-1} = \frac{1}{1} = 1$
$f(2) = \frac{2}{2(2)-1} \Rightarrow \frac{2}{4-1} = \frac{2}{3}$
$f(3) = \frac{3}{2(3)-1} = \frac{3}{6-1} = \frac{3}{5}$
$f(4) = \frac{4}{2(4)-1} = \frac{4}{8-1} = \frac{4}{7}$
$f(5) = \frac{5}{2(5)-1} = \frac{5}{10-1} = \frac{5}{9}$
$f(6) = \frac{6}{2(6)-1} = \frac{6}{12-1} = \frac{6}{11}$

College Algebra – Chapter 11
Lesson 1, Day 1

Describe the pattern, write the next term, and write a rule for the nth term of the sequence.

11. 1, 6, 11, 16, . . .

$a_n = 5n - 4$

$a_5 = 5(5) - 4 \Rightarrow 25 - 4 = 21$

Arithmetic

12. 1, 2, 4, 8, . . .

Geometric

$a_n = 2^{n-1}$

$a_5 = 2^{5-1} \Rightarrow 2^4 = 16$

13. 3.1, 3.8, 4.5, 5.2, . . .

$a_n = 0.7n + 2.4$

$a_5 = 0.7(5) + 2.4 = 5.9$

14. 9, 16.8, 24.6, 32.4, . . .

$a_n = 7.8n + 1.2$

$a_5 = 7.8(5) + 1.2 = 40.2$

15. -4, 8, -12, 16, . . .

$a_n = (-1)^n 4n$

$a_5 = (-1)^5 4(5)$

$\Rightarrow -1(20)$

$= -20$

16. 2, 9, 28, 65, . . .

$a_n = n^3 + 1$

$a_5 = (5)^3 + 1$

$= 125 + 1$

126

College Algebra – Chapter 11
Lesson 1, Day 2

Simplify the factorial expression.

1. $\frac{3!}{5!}$

$\frac{3!}{5 \cdot 4 \cdot 3!} = \boxed{\frac{1}{20}}$

2. $\frac{8!}{10!}$

$\frac{8!}{10 \cdot 9 \cdot 8!} = \boxed{\frac{1}{90}}$

3. $\frac{12!}{11!}$

$\frac{12 \cdot 11!}{11!} = \boxed{12}$

4. $\frac{20!}{18!}$

$\frac{20 \cdot 19 \cdot 18!}{18!} = \boxed{380}$

5. $\frac{n!}{(n+1)!}$

$\frac{n!}{(n+1) \cdot (n+1-1)!} = \boxed{\frac{1}{n+1}}$

6. $\frac{(n-1)!}{(n-2)!}$

$\frac{(n-1) \cdot (n-1-1)!}{(n-2)!} = \boxed{n-1}$

7. $\frac{(2n+1)!}{(2n)!}$

$\frac{(2n+1) \cdot (2n+1-1)!}{(2n)!} = \boxed{2n+1}$

8. $\frac{(2n+1)!}{(2n-1)!}$

$\frac{(2n+1) \cdot (2n+1-1) \cdot (2n-1)!}{(2n-1)!}$

$(2n+1) \cdot (2n) = \boxed{4n^2+2n}$

College Algebra – Chapter 11
Lesson 1, Day 3

Write the series using summation notation.

1. $7 + 10 + 13 + 16 + 19$

$a_n = 3i + 4$

$$\sum_{i=1}^{5} 3i + 4$$

2. $5 + 11 + 17 + 23 + 29$

$a_n = 6i - 1$

$$\sum_{i=1}^{5} 6i - 1$$

3. $4 + 7 + 12 + 19 + \ldots$

$a_n = i^2 + 3$

$$\sum_{i=1}^{\infty} i^2 + 3$$

4. $-1 + 2 + 7 + 9 + \ldots$

$a_n = i^2 - 2$

$$\sum_{i=1}^{\infty} i^2 - 2$$

5. $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots$

$a_n = \frac{1}{3^i}$

$$\sum_{i=1}^{\infty} \frac{1}{3^i}$$

6. $-3 + 4 - 5 + 6 - 7$

$a_n = (-1)^i (i + 2)$

$$\sum_{i=1}^{5} (-1)^i (i + 2)$$

7. $-2 + 4 - 8 + 16 - 32$

$a_n = (-2)^i$

$$\sum_{i=1}^{5} (-2)^i$$

8. $\frac{1}{4} + \frac{2}{5} + \frac{3}{6} + \frac{4}{7} + \ldots$

$a_n = \frac{i}{i+3}$

$$\sum_{i=1}^{\infty} \frac{i}{i+3}$$

College Algebra – Chapter 11
Lesson 1, Day 3
Find the sum.

9. $\sum_{i=1}^{6} 2i$

$2(1)+2(2)+2(3)+2(4)+2(5)+2(6)$
$2+4+6+8+10+12$
42

10. $\sum_{i=1}^{5} 7i$

$7(1)+7(2)+7(3)+7(4)+7(5)$
$7+14+21+28+35$
105

11. $\sum_{n=0}^{4} n^3$

$(0)^3+(1)^3+(2)^3+(3)^3+(4)^3$
$0+1+8+27+64$
100

12. $\sum_{k=1}^{4} 3k^2$

$3(1)^2+3(2)^2+3(3)^2+3(4)^2$
$3(1)+3(4)+3(9)+3(16)$
$3+12+27+48$
90

13. $\sum_{k=1}^{6}(5k-2)$

$(5(1)-2)+(5(2)-2)+(5(3)-2)+(5(4)-2)+(5(5)-2)+(5(6)-2)$
$(5-2)+(10-2)+(15-2)+(20-2)+(25-2)+(30-2)$
$3+8+13+18+23+28$
82

14. $\sum_{n=1}^{5}(n^2-1)$

$(1^2-1)+(2^2-1)+(3^2-1)+(4^2-1)+(5^2-1)$
$(1-1)+(4-1)+(9-1)+(16-1)+(25-1)$
$0+3+8+15+24$
50

15. $\sum_{i=2}^{8} \frac{2}{i}$

$\frac{2}{2}+\frac{2}{3}+\frac{2}{4}+\frac{2}{5}+\frac{2}{6}+\frac{2}{7}+\frac{2}{8}$

$\frac{481}{140}$

16. $\sum_{k=4}^{6} \frac{k}{k+1}$

$\frac{4}{(4+1)}+\frac{5}{(5+1)}+\frac{6}{(6+1)}$

$\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$

$\frac{523}{210}$

College Algebra – Chapter 11
Lesson 2, Day 1

Tell whether the sequence in arithmetic. Explain your reasoning.

1. 1, -1, -3, -5, -7, . . .

$-1-1=-2$
$-3-(-1)=-2$
$-5-(-3)=-2$
$-7-(-5)=-2$

Common Difference

2. 12, 6, 0, -6, -12, . . .

$6-12=-6$
$0-6=-6$
$-6-(-6)=-6$
$-12-(-6)=-6$

Common Difference

3. 5, 8, 13, 20, 29, . . .

$8-5=3$
$13-8=5$

Not Arithmetic

4. 3, 5, 9, 15, 23, . . .

$5-3=2$
$9-5=4$

Not Arithmetic

5. 36, 18, 9, $\frac{9}{2}$, $\frac{9}{4}$, . . .

$18-36=-18$
$9-18=-9$

Not Arithmetic

6. 81, 27, 9, 3, 1, . . .

$27-81=-54$
$9-27=-18$

Not Arithmetic

Write a rule for the arithmetic sequence with the given description.

7. The firs term is -3 and each term is 6 less than the previous term.

$d=-6 \quad a_1=-3$
$a_n=-3+(n-1)(-6)$
$=-3-6n+6$
$\rightarrow \boxed{-6n+3}$

8. The first term is 7 and each term is 5 more than the previous term.

$a_1=7 \quad d=5$
$a_n=7+(n-1)(5)$
$\Rightarrow 7+5n-5$
$=\boxed{5n+2}$

College Algebra – Chapter 11
Lesson 2, Day 1

Write a rule for the nth term of the sequence. Then find a_{20}.

9. 12, 20, 28, 36, . . .

$20-12=8$ ✓ $a_1=12$ $d=8$
$28-20=8$

$a_n = 12+(n-1)(8)$
$\Rightarrow 12+8n-8$
$= \boxed{8n+4}$

$a_{20} = 8(20)+4$
$\Rightarrow 160+4$
$\boxed{164}$

10. 7, 12, 17, 22, . . .

$12-7=5$ $a_1=7$ $d=5$
$17-12=5$ ✓

$a_n = 7+(n-1)(5)$
$\Rightarrow 7+5n-5$
$= \boxed{5n+2}$

$a_{20} = 5(20)+2$
$= 100+2 = \boxed{102}$

11. 51, 48, 45, 42, . . .

$48-51=-3$ $a_1=51$ $d=-3$
$45-48=-3$ ✓

$a_n = 51+(n-1)(-3)$
$\Rightarrow 51-3n+3$
$= \boxed{-3n+54}$

$a_{20} = -3(20)+54$
$\Rightarrow -60+54 = \boxed{-6}$

12. 86, 79, 72, 65, . . .

$79-86=-7$ $a_1=86$ $d=-7$
$72-79=-7$ ✓

$a_n = 86+(n-1)(-7)$
$\Rightarrow 86-7n+7$
$= \boxed{-7n+93}$

$a_{20} = -7(20)+93$
$\Rightarrow -140+93 = \boxed{-47}$

13. $-1, -\frac{1}{3}, \frac{1}{3}, 1, \ldots$

$-\frac{1}{3}+\left(+\frac{2}{3}\right)=\frac{2}{3}$ $a_1=-1$ $d=\frac{2}{3}$
$\frac{1}{3}-\left(-\frac{1}{3}\right)=\frac{2}{3}$ ✓

$a_n = -1+(n-1)\left(\frac{2}{3}\right)$
$\Rightarrow -1+\frac{2}{3}n-\frac{2}{3}$
$\Rightarrow \frac{2}{3}n-\frac{3}{3}-\frac{2}{3}$
$= \boxed{\frac{2}{3}n-\frac{5}{3}}$

$a_{20} = \frac{2}{3}(20)-\frac{5}{3}$
$= \frac{40}{3}-\frac{5}{3}$
$= \boxed{\frac{35}{3}}$

14. $-2, -\frac{5}{4}, -\frac{1}{2}, \frac{1}{4}, \ldots$

$-\frac{5}{4}+\left(+\frac{3}{4}\right)=\frac{3}{4}$ $a_1=-2$ $d=\frac{3}{4}$
$-\frac{2}{4}-\left(-\frac{5}{4}\right)=\frac{3}{4}$

$a_n = -2+(n-1)\left(\frac{3}{4}\right)$
$= -2+\frac{3}{4}n-\frac{3}{4}$
$= \frac{3}{4}n-\frac{3}{4}-\frac{8}{4}$
$= \boxed{\frac{3}{4}n-\frac{11}{4}}$

$a_{20} = \frac{3}{4}(20)-\frac{11}{4}$
$= \frac{60}{4}-\frac{11}{4}$
$= \boxed{\frac{49}{4}}$

Write a rule for the nth term of the sequence.

15. $a_{11}=43, d=5.$

$a_{11} = a_1+(11-1)(5)$
$43 = a_1+(10)(5)$
$43 = a_1+50$
$-50 \quad -50$
$-7 = a_1$

$a_n = -7+(n-1)(5)$
$= -7+5n-5$
$= \boxed{5n-12}$

16. $a_{13}=42, d=4$

$a_{13} = a_1+(13-1)(4)$
$42 = a_1+(12)(4)$
$42 = a_1+48$
$-48 \quad -48$
$-6 = a_1$

$a_n = -6+(n-1)(4)$
$= -6+4n-4$
$= \boxed{4n-10}$

17. $a_{20}=-27, d=-2$

$a_{20} = a_1+(20-1)(-2)$
$-27 = a_1+(19)(-2)$
$-27 = a_1-38$
$+38 \quad +38$
$11 = a_1$

$a_n = 11+(n-1)(-2)$
$\Rightarrow 11-2n+2$
$= \boxed{-2n+13}$

18. $a_{15}=-35, d=-3$

$a_{15} = a_1+(15-1)(-3)$
$-35 = a_1+(14)(-3)$
$-35 = a_1-42$
$+42 \quad +42$
$7 = a_1$

$a_n = 7+(n-1)(-3)$
$\Rightarrow 7-3n+3$
$= \boxed{-3n+10}$

College Algebra – Chapter 11
Lesson 2, Day 2

Write a rule for the nth term of the arithmetic sequence.

1. $a_5 = 41, a_{10} = 96$

$a_{10} = a_1 + (10-1)(d) \rightarrow 96 = a_1 + 9d$
$a_5 = a_1 + (5-1)(d) \rightarrow 41 = a_1 + 4d \xrightarrow{-1} -41 = -a_1 - 4d$

$\frac{55}{5} = \frac{5d}{5}$

$11 = d$

$96 = a_1 + 9(11)$
$96 = a_1 + 99$
$-99 \quad -99$

$-3 = a_1$

$a_n = -3 + (n-1)(11)$
$\Rightarrow -3 + 11n - 11$

$a_n = 11n - 14$

2. $a_7 = 58, a_{11} = 94$

$a_{11} = a_1 + (11-1)d \rightarrow 94 = a_1 + 10d \rightarrow 94 = a_1 + 10d$
$a_7 = a_1 + (7-1)d \rightarrow 58 = a_1 + 6d \xrightarrow{-1} -58 = -a_1 - 6d$

$\frac{36}{4} = \frac{4d}{4}$

$9 = d$

$94 = a_1 + 10(9)$
$94 = a_1 + 90$
$-90 \quad -90$

$4 = a_1$

$a_n = 4 + (n-1)(9)$
$\Rightarrow 4 + 9n - 9$

$a_n = 9n - 5$

3. $a_6 = -8, a_{15} = -62$

$a_{15} = a_1 + (15-1)d \rightarrow -62 = a_1 + 14d \rightarrow -62 = a_1 + 14d$
$a_6 = a_1 + (6-1)d \rightarrow -8 = a_1 + 5d \xrightarrow{-1} 8 = -a_1 - 5d$

$\frac{-54}{9} = \frac{9d}{9}$

$-6 = d$

$-62 = a_1 + 14(-6)$
$-62 = a_1 - 84$
$+84 \quad +84$

$22 = a_1$

$a_n = 22 + (n-1)(-6)$
$\Rightarrow 22 - 6n + 6$
$= -6n + 28$

4. $a_8 = -15, a_{17} = -78$

$a_{17} = a_1 + (17-1)d \rightarrow -78 = a_1 + 16d \rightarrow -78 = a_1 + 16d$
$a_8 = a_1 + (8-1)d \rightarrow -15 = a_1 + 7d \xrightarrow{-1} 15 = -a_1 - 7d$

$\frac{-63}{9} = \frac{9d}{9}$

$-7 = d$

$-78 = a_1 + 16(-7)$
$-78 = a_1 - 112$
$+112 \quad +112$

$34 = a_1$

$a_n = 34 + (n-1)(-7)$
$= 34 - 7n + 7$
$= -7n + 41$

5. $a_{18} = -59, a_{21} = -71$

$a_{21} = a_1 + (21-1)d \rightarrow -71 = a_1 + 20d \rightarrow -71 = a_1 + 20d$
$a_{18} = a_1 + (18-1)d \rightarrow -59 = a_1 + 17d \xrightarrow{-1} 59 = -a_1 - 17d$

$\frac{-12}{3} = \frac{3d}{3}$

$-4 = d$

$-71 = a_1 + 20(-4)$
$-71 = a_1 - 80$
$+80 \quad +80$

$9 = a_1$

$a_n = 9 + (n-1)(-4)$
$= 9 - 4n + 4$
$= -4n + 13$

6. $a_{12} = -38, a_{19} = -73$

$a_{19} = a_1 + (19-1)d \rightarrow -73 = a_1 + 18d \rightarrow -73 = a_1 + 18d$
$a_{12} = a_1 + (12-1)d \rightarrow -38 = a_1 + 11d \xrightarrow{-1} 38 = -a_1 - 11d$

$\frac{-35}{7} = \frac{7d}{7}$

$-5 = d$

$-73 = a_1 + 18(-5)$
$-73 = a_1 - 90$
$+90 \quad +90$

$17 = a_1$

$a_n = 17 + (n-1)(-5)$
$= 17 - 5n + 5$
$= -5n + 22$

7. $a_8 = 12, a_{16} = 22$

$a_{16} = a_1 + (16-1)d \rightarrow 22 = a_1 + 15d \rightarrow 22 = a_1 + 15d$
$a_8 = a_1 + (8-1)d \rightarrow 12 = a_1 + 7d \xrightarrow{-1} -12 = -a_1 - 7d$

$\frac{10}{8} = \frac{8d}{8}$

$\frac{10}{8} = d$

$\frac{5}{4} = d$

$22 = a_1 + 15\left(\frac{5}{4}\right)$
$22 = a_1 + \frac{75}{4}$
$-\frac{75}{4} \quad -\frac{75}{4}$

$\frac{88}{4} - \frac{75}{4} = a_1$

$\frac{13}{4} = a_1$

$a_n = \frac{13}{4} + (n-1)\left(\frac{5}{4}\right)$
$= \frac{13}{4} + \frac{5}{4}n - \frac{5}{4}$
$= \frac{5}{4}n + \frac{8}{4}$
$= \frac{5}{4}n + 2$

8. $a_{12} = 9, a_{27} = 15$

$a_{27} = a_1 + (27-1)d \rightarrow 15 = a_1 + 26d \rightarrow 15 = a_1 + 26d$
$a_{12} = a_1 + (12-1)d \rightarrow 9 = a_1 + 11d \xrightarrow{-1} -9 = -a_1 - 11d$

$\frac{6}{15} = \frac{15d}{15}$

$\frac{6}{15} = d$

$\frac{2}{5} = d$

$15 = a_1 + 26\left(\frac{2}{5}\right)$
$15 = a_1 + \frac{52}{5}$
$-\frac{52}{5} \quad -\frac{52}{5}$

$\frac{75}{5} - \frac{52}{5} = a_1$

$\frac{23}{5} = a_1$

$a_n = \frac{23}{5} + (n-1)\left(\frac{2}{5}\right)$
$= \frac{23}{5} + \frac{2}{5}n - \frac{2}{5}$
$= \frac{2}{5}n + \frac{21}{5}$

College Algebra – Chapter 11
Lesson 2, Day 2

Find the sum.

9. $\sum_{i=1}^{20}(2i-3)$.

$a_1 = 2(1)-3 = 2-3 = -1$ $\quad a_{20} = 2(20)-3 = 40-3 = 37$

$S_{20} = 20\left(\frac{-1+37}{2}\right)$

$= 20\left(\frac{36}{2}\right)$

$= 20(18) =$ **360**

10. $\sum_{i=1}^{26}(4i+7)$

$a_1 = 4(1)+7 = 4+7 = 11$ $\quad a_{26} = 4(26)+7 = 104+7 = 111$

$S_{26} = 26\left(\frac{11+111}{2}\right)$

$= 26\left(\frac{122}{2}\right) \Rightarrow 26(61)$

1,586

11. $\sum_{i=1}^{33}(6-2i)$

$a_1 = 6-2(1) = 6-2 = 4$ $\quad a_{33} = 6-2(33) = 6-66 = -60$

$S_{33} = 33\left(\frac{4+(-60)}{2}\right) \Rightarrow 33\left(\frac{-56}{2}\right)$

$\Rightarrow 33(-28)$

-924

12. $\sum_{i=1}^{31}(-3-4i)$

$a_1 = -3-4(1) = -3-4 = -7$ $\quad a_{31} = -3-4(31) = -3-124 = -127$

$S_{31} = 31\left(\frac{-7+(-127)}{2}\right) \Rightarrow 31\left(\frac{-134}{2}\right)$

$\Rightarrow 31(-67)$

-2,077

13. $\sum_{i=1}^{41}(-2.3+0.1i)$

$a_1 = -2.3+0.1(1) = -2.3+0.1 = -2.2$ $\quad a_{41} = -2.3+0.1(41) = -2.3+4.1 = 1.8$

$S_{41} = 41\left(\frac{-2.2+1.8}{2}\right) \Rightarrow 41\left(\frac{-0.4}{2}\right)$

$\Rightarrow 41(-0.2)$

$= -8.2$

14. $\sum_{i=1}^{39}(-4.1+0.4i)$

$a_1 = -4.1+0.4(1) = -4.1+0.4 = -3.7$ $\quad a_{39} = -4.1+0.4(39) = -4.1+15.6 = 11.5$

$S_{39} = 39\left(\frac{-3.7+11.5}{2}\right) \Rightarrow 39\left(\frac{7.8}{2}\right)$

$\Rightarrow 39(3.9) =$ **152.1**

15. The first 19 terms of the sequence 9, 2, -5, -12, . . .

$2-9 = -7$, $-5-2 = -7$ $\quad d = -7$ $\quad a_1 = 9$

$a_n = 9+(n-1)(-7)$

$= 9-7n+7$

$= -7n+16$

$a_1 = 9$ $\quad a_{19} = -7(19)+16 = -133+16 = -117$

$S_{19} = 19\left(\frac{9-117}{2}\right)$

$= 19\left(\frac{-108}{2}\right)$

$= 19(-54)$

-1,026

16. The first 22 terms of the sequence 17, 9, 1, -7, . . .

$9-17 = -8$, $1-9 = -8$ $\quad a_1 = 17$ $\quad d = -8$

$a_n = 17+(n-1)(-8)$

$= 17-8n+8$

$= -8n+25$

$a_1 = 17$ $\quad a_{22} = -8(22)+25 = -176+25 = -151$

$S_{22} = 22\left(\frac{17-151}{2}\right)$

$\Rightarrow 22\left(\frac{-134}{2}\right) \Rightarrow 22(-67)$

$= -1,474$

College Algebra – Chapter 11
Lesson 3, Day 1

Tell whether the sequence is geometric. Explain your reasoning.

1. 96, 48, 24, 12, 6, . . .

$\frac{48}{96} = \frac{1}{2}$

$\frac{24}{48} = \frac{1}{2}$

$\frac{12}{24} = \frac{1}{2}$

$\frac{6}{12} = \frac{1}{2}$

Geometric
Common Ratio

2. 729, 243, 81, 27, 9, . . .

$\frac{243}{729} = \frac{1}{3}$

$\frac{81}{243} = \frac{1}{3}$

$\frac{27}{81} = \frac{1}{3}$

$\frac{9}{27} = \frac{1}{3}$

Geometric
Common Ratio

3. 2, 4, 6, 8, 10, . . .

$\frac{4}{2} = 2$

$\frac{6}{4} = \frac{3}{2}$

No Common Ratio

4. 5, 20, 35, 50, 65, . . .

$\frac{20}{5} = 4$

$\frac{35}{20} = \frac{7}{4}$

No Common Ratio

5. 0.2, 3.2, -12.8, 51.2, -204.8, . . .

$\frac{3.2}{0.2} = 16$

$\frac{-12.8}{3.2} = -4$

No Common Ratio

6. 0.3, -1.5, 7.5, -37.5, 187.5, . . .

$\frac{-1.5}{0.3} = -5$

$\frac{7.5}{-1.5} = -5$

$\frac{-37.5}{7.5} = -5$

$\frac{187.5}{-37.5} = -5$

Geometric
Common Ratio

7. $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \frac{1}{162}, \ldots$

$\frac{\frac{1}{6}}{\frac{1}{2}} \Rightarrow \frac{1}{6} \cdot \frac{2}{1} = \frac{2}{6} = \frac{1}{3}$

$\frac{\frac{1}{18}}{\frac{1}{6}} \Rightarrow \frac{1}{18} \cdot \frac{6}{1} = \frac{6}{18} = \frac{1}{3}$

$\frac{\frac{1}{54}}{\frac{1}{18}} \Rightarrow \frac{1}{54} \cdot \frac{18}{1} = \frac{18}{54} = \frac{1}{3}$

$\frac{\frac{1}{162}}{\frac{1}{54}} = \frac{1}{162} \cdot \frac{54}{1} = \frac{54}{162} = \frac{1}{3}$

Geometric
Common Ratio

College Algebra – Chapter 11
Lesson 3, Day 1

Write a rule for the nth term of the sequence. Then find a_7.

8. 4, 20, 100, 500, . . .

$\frac{20}{4}=5$ $\frac{100}{20}=5$ $a_1=4$ $r=5$

$a_n=4(5)^{n-1}$

$a_7=4(5)^{7-1} \Rightarrow 4(5)^6$

$\Rightarrow 4(15625)=62,500$

9. 6, 24, 96, 384, . . .

$\frac{24}{6}=4$ $\frac{96}{24}=4$ $a_1=6$ $r=4$

$a_n=6(4)^{n-1}$

$a_7=6(4)^{7-1} \Rightarrow 6(4)^6$

$\Rightarrow 6(4096)=24,576$

10. 112, 56, 28, 14, . . .

$\frac{56}{112}=\frac{1}{2}$ $\frac{28}{56}=\frac{1}{2}$ $a_1=112$ $r=\frac{1}{2}$

$a_n=112\left(\frac{1}{2}\right)^{n-1}$

$a_7=112\left(\frac{1}{2}\right)^{7-1} \Rightarrow 112\left(\frac{1}{2}\right)^6$

$\Rightarrow 112\left(\frac{1}{64}\right)=\frac{112}{64}=\frac{7}{4}$

11. 375, 75, 15, 3, . . .

$\frac{75}{375}=\frac{1}{5}$ $\frac{15}{75}=\frac{1}{5}$ $a_1=375$ $r=\frac{1}{5}$

$a_n=375\left(\frac{1}{5}\right)^{n-1}$

$a_7=375\left(\frac{1}{5}\right)^{7-1} \Rightarrow 375\left(\frac{1}{5}\right)^6$

$\Rightarrow 375\left(\frac{1}{15625}\right) \Rightarrow \frac{375}{15625}=\frac{3}{125}$

12. 4, 6, 9, $\frac{27}{2}$, . . .

$\frac{6}{4}=\frac{3}{2}$ $\frac{9}{6}=\frac{3}{2}$ $a_1=4$ $r=\frac{3}{2}$

$a_n=4\left(\frac{3}{2}\right)^{n-1}$

$a_7=4\left(\frac{3}{2}\right)^{7-1} \Rightarrow 4\left(\frac{3}{2}\right)^6$

$\Rightarrow 4\left(\frac{729}{64}\right)=\frac{729}{16}$

13. 2, $\frac{3}{2}$, $\frac{9}{8}$, $\frac{27}{32}$, . . .

$\frac{3}{2} \Rightarrow \frac{3}{2}\cdot\frac{1}{2}=\frac{3}{4}$

$\frac{9}{8} \Rightarrow \frac{3}{2}\cdot\frac{3}{4}=\frac{3}{4}$

$a_1=2$ $r=\frac{3}{4}$

$a_n=2\left(\frac{3}{4}\right)^{n-1}$

$a_7=2\left(\frac{3}{4}\right)^{7-1} \Rightarrow 2\left(\frac{3}{4}\right)^6$

$\Rightarrow 2\left(\frac{729}{4096}\right) \Rightarrow \frac{729}{2048}$

Write a rule for the nth term of the geometric sequence.

14. $a_3=4, r=2$

$a_3=a_1(2)^{3-1}$

$4=a_1(2)^2$

$\frac{4}{4}=\frac{a_1 4}{4}$

$1=a_1$

$a_n=1(2)^{n-1}$

15. $a_3=27, r=3$

$a_3=a_1(3)^{3-1}$

$27=a_1(3)^2$

$\frac{27}{9}=\frac{a_1 9}{9}$

$3=a_1$

$a_n=3(3)^{n-1}$

16. $a_2=30, r=\frac{1}{2}$

$a_2=a_1\left(\frac{1}{2}\right)^{2-1}$

$30=a_1\left(\frac{1}{2}\right)^1\left(\frac{2}{1}\right)$

$60=a_1$

$a_n=60\left(\frac{1}{2}\right)^{n-1}$

17. $a_2=64, r=\frac{1}{4}$

$a_2=a_1\left(\frac{1}{4}\right)^{2-1}$

$(4)64=a_1\left(\frac{1}{4}\right)^1(4)$

$256=a_1$

$a_n=256\left(\frac{1}{4}\right)^{n-1}$

College Algebra – Chapter 11

Lesson 3, Day 2

Write a rule for the nth term of the geometric sequence.

1. $a_2 = 28, a_5 = 1792$

$a_5 = a_1 r^{5-1} \rightarrow 1792 = a_1 r^4$

$a_2 = a_1 r^{2-1} \rightarrow \frac{28}{r} = \frac{a_1 r}{r}$

$a_1 = \frac{28}{r}$

$1792 = \frac{28}{r}(r^4)$

$\frac{1792}{28} = \frac{28r^3}{28}$

$\sqrt[3]{64} = \sqrt[3]{r^3}$

$4 = r$

$a_1 = \frac{28}{4} = 7$

$a_n = 7(4)^{n-1}$

2. $a_1 = 11, a_4 = 88$

$a_4 = a_1 r^{4-1} \rightarrow 88 = a_1 r^3$

$a_1 = a_1 r^{1-1} \rightarrow 11 = a_1$

$\frac{88}{11} = \frac{11r^3}{11}$

$\sqrt[3]{8} = \sqrt[3]{r^3}$

$2 = r$

$a_n = 11(2)^{n-1}$

3. $a_1 = -6, a_5 = -486$

$a_5 = a_1 r^{5-1} \rightarrow -486 = a_1 r^4$

$a_1 = a_1 r^{1-1} \rightarrow -6 = a_1$

$\frac{-486}{-6} = \frac{-6r^4}{-6}$

$\sqrt[4]{81} = \sqrt[4]{r^4}$

$\pm 3 = r$

$a_n = -6(3)^{n-1}$ OR $a_n = -6(-3)^{n-1}$

4. $a_2 = -10, a_6 = -6250$

$a_6 = a_1 r^{6-1} \rightarrow -6250 = a_1 r^5$

$a_2 = a_1 r^{2-1} \rightarrow -10 = \frac{a_1 r}{r} \Rightarrow a_1 = \frac{-10}{r}$

$-6250 = \left(\frac{-10}{r}\right) r^5$

$\frac{-6250}{-10} = \frac{-10r^4}{-10}$

$\sqrt[4]{625} = \sqrt[4]{r^4}$

$\pm 5 = r$

$a_1 = \frac{-10}{5} = -2$ $\quad a_1 = \frac{-10}{-5} = 2$

$a_n = -2(5)^{n-1}$ or $a_n = 2(-5)^{n-1}$

5. $a_2 = 64, a_4 = 1$

$a_4 = a_1 r^{4-1} \rightarrow 1 = a_1 r^3$

$a_2 = a_1 r^{2-1} \rightarrow \frac{64}{r} = \frac{a_1 r}{r} \Rightarrow a_1 = \frac{64}{r}$

$1 = \left(\frac{64}{r}\right) r^3$

$\frac{1}{64} = \frac{64r^3}{64}$

$\sqrt{\frac{1}{64}} = \sqrt{r^3}$

$\pm \frac{1}{8} = r$

$a_1 = \frac{64}{1/8} \Rightarrow \frac{64}{1} \cdot \frac{8}{1} \Rightarrow 512$

$a_1 = \frac{64}{-1/8} \Rightarrow 64 \cdot -8 = -512$

$a_n = 512\left(\frac{1}{8}\right)^{n-1}$ or $a_n = -512\left(\frac{1}{8}\right)^{n-1}$

6. $a_1 = 1, a_2 = 49$

$a_2 = a_1 r^{2-1} \rightarrow 49 = a_1 r$

$a_1 = a_1 r^{1-1} \rightarrow 1 = a_1$

$49 = 1r$

$49 = r$

$a_n = 1(49)^{n-1}$

Find the sum.

7. $\sum_{i=1}^{9} 6(7)^{i-1}$

$a_1 = 6 \quad r = 7$

$S_9 = 6\left(\frac{1-7^9}{1-7}\right) \Rightarrow 6\left(\frac{1-40353607}{-6}\right)$

$\Rightarrow 6\left(\frac{-40353606}{-6}\right)$

$= 40353606$

8. $\sum_{i=1}^{10} 7(4)^{i-1}$

$a_1 = 7 \quad r = 4$

$S_{10} = 7\left(\frac{1-4^{10}}{1-4}\right) \Rightarrow 7\left(\frac{1-1,048,576}{-3}\right)$

$\Rightarrow 7\left(\frac{-1,048,575}{-3}\right) \Rightarrow 7(349,525)$

$= 2,446,675$

9. $\sum_{i=1}^{10} \left(\frac{4}{3}\right)^{i-1}$

$a_1 = 1 \quad r = \frac{4}{3}$

$S_{10} = 1\left(\frac{1-\frac{4}{3}^{10}}{1-\frac{4}{3}}\right) \Rightarrow 1\left(\frac{1-\frac{1048576}{59049}}{\frac{3}{3}-\frac{4}{3}}\right)$

$\Rightarrow 1\left(\frac{\frac{59049}{59049}-\frac{1048576}{59049}}{-\frac{1}{3}}\right) \Rightarrow 1\left(\frac{-989,527}{59049} \div -\frac{1}{3}\right)$

$\Rightarrow \frac{-989,527}{59049} \cdot -\frac{3}{1} \Rightarrow \frac{989527}{19683}$

10. $\sum_{i=1}^{8} 5\left(\frac{1}{3}\right)^{i-1}$

$a_1 = 5 \quad r = \frac{1}{3}$

$S_8 = 5\left(\frac{1-\frac{1}{3}^8}{1-\frac{1}{3}}\right) \Rightarrow 5\left(\frac{1-\frac{1}{6561}}{\frac{3}{3}-\frac{1}{3}}\right) \Rightarrow 5\left(\frac{\frac{6561}{6561}-\frac{1}{6561}}{\frac{2}{3}}\right)$

$\Rightarrow 5\left(\frac{\frac{6560}{6561}}{\frac{2}{3}}\right) \Rightarrow 5\left(\frac{3280}{\frac{6560}{6561}} \cdot \frac{2}{3}\right) = 5\left(\frac{3280}{2187}\right)$

$= \frac{16400}{2187}$

College Algebra – Chapter 11

Lesson 3, Day 2

11. $\sum_{i=0}^{8} 8\left(-\frac{2}{3}\right)^i$

$a_1 = 8\left(-\frac{2}{3}\right)^1 \Rightarrow 8\left(-\frac{2}{3}\right) = -\frac{16}{3}$ $\quad r = -\frac{2}{3}$

$a_8 = -\frac{16}{3}\left(\frac{1-\left(-\frac{2}{3}\right)^8}{1-\left(-\frac{2}{3}\right)}\right) \Rightarrow -\frac{16}{3}\left(\frac{1-\frac{256}{6561}}{\frac{3}{3}+\frac{2}{3}}\right) \Rightarrow -\frac{16}{3}\left(\frac{\frac{6561}{6561}-\frac{256}{6561}}{\frac{5}{3}}\right)$

$\Rightarrow -\frac{16}{3}\left(\frac{\frac{6305}{6561}}{\frac{5}{3}}\right) \Rightarrow -\frac{16}{3}\left(\frac{6305}{6561}\cdot\frac{3}{5}\right) \Rightarrow -\frac{16}{3}\left(\frac{1261}{2187}\right)$

$= \boxed{-\frac{20176}{6561}}$

12. $\sum_{i=0}^{9} 9\left(-\frac{3}{4}\right)^i$

$a_1 = 9\left(-\frac{3}{4}\right)^1 \Rightarrow -\frac{27}{4}$ $\quad r = -\frac{3}{4}$

$a_9 = -\frac{27}{4}\left(\frac{1-\left(-\frac{3}{4}\right)^9}{1-\left(-\frac{3}{4}\right)}\right) \Rightarrow -\frac{27}{4}\left(\frac{1+\frac{19683}{262144}}{\frac{4}{4}+\frac{3}{4}}\right)$

$\Rightarrow -\frac{27}{4}\left(\frac{\frac{281887}{262144}}{\frac{7}{4}}\right) \Rightarrow -\frac{27}{4}\left(\frac{281887}{262144}\cdot\frac{4}{7}\right)$

$\boxed{-\frac{1087047}{262144}}$

13. The first 8 terms of the geometric sequence -12, -48, -192, -768, . . .

$a_1 = -12 \quad r = 4$

$a_8 = -12\left(\frac{1-4^8}{1-4}\right) \Rightarrow -12\left(\frac{1-65536}{-3}\right) \Rightarrow -12\left(\frac{-65535}{-3}\right) \Rightarrow \boxed{-262,140}$

14. The first 9 terms of the geometric sequence -14, -42, -126, -378, . . .

$a_1 = -14 \quad r = 3$

$a_9 = -14\left(\frac{1-3^9}{1-3}\right) \Rightarrow -14\left(\frac{1-19683}{-2}\right) \Rightarrow -14\left(\frac{-19682}{-2}\right) = \boxed{-137,774}$

College Algebra – Chapter 11

Lesson 4

Consider the infinite geometric series. Find and graph the partial sums S_n for n = 1, 2, 3, 4, and 5. The describe what happens to S_n as n increases.

1. $\frac{1}{2}+\frac{1}{6}+\frac{1}{18}+\frac{1}{54}+\frac{1}{162}+\dots$

$S_1 = 0.5$
$S_2 = 0.67$
$S_3 \approx 0.72$
$S_4 \approx 0.74$
$S_5 \approx 0.75$

Appears to approach 0.76

2. $\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}+\dots$

$S_1 \approx 0.67$
$S_2 = 1$
$S_3 \approx 1.17$
$S_4 \approx 1.25$
$S_5 \approx 1.29$

Appears to approach $\frac{4}{3}$ or 1.33

3. $4+\frac{12}{5}+\frac{36}{25}+\frac{108}{125}+\frac{324}{625}+\dots$

$S_1 = 4$
$S_2 = 6.4$
$S_3 = 7.87$
$S_4 \approx 8.70$
$S_5 \approx 9.22$

Appears to Approach 10

4. $2+\frac{2}{6}+\frac{2}{36}+\frac{2}{216}+\frac{2}{1296}+\dots$

$S_1 = 2$
$S_2 \approx 2.33$
$S_3 \approx 2.39$
$S_4 \approx 2.40$
$S_5 \approx 2.40$

Appears to Approach 2.40

Find the sum of the infinite geometric series, if it exists.

5 $\sum_{n=1}^{\infty}(\frac{1}{5})^{n-1}$.

$a_1 = 1 \quad r = \frac{1}{5}$

$S = \frac{1}{1-\frac{1}{5}} \Rightarrow \frac{1}{\frac{5}{5}-\frac{1}{5}} \Rightarrow \frac{1}{4/5} \Rightarrow 1 \cdot \frac{5}{4} = \boxed{\frac{5}{4}}$

6. $\sum_{k=1}^{\infty} -6(\frac{3}{2})^{k-1}$

$a_1 = -6 \quad r = \frac{3}{2}$

$|\frac{3}{2}| \geq 1$, sum does not exist

7. $\sum_{k=1}^{\infty} \frac{11}{3}(\frac{3}{8})^{k-1}$

$a_1 = \frac{11}{3} \quad r = \frac{3}{8}$

$S = \frac{\frac{11}{3}}{1-\frac{3}{8}} \Rightarrow \frac{\frac{11}{3}}{\frac{8}{8}-\frac{3}{8}} = \frac{\frac{11}{3}}{\frac{5}{8}} \Rightarrow \frac{11}{3} \cdot \frac{8}{5}$

$\boxed{\frac{88}{15}}$

8. $\sum_{i=1}^{\infty} \frac{2}{5}(\frac{5}{3})^{i-1}$

$a_1 = \frac{2}{5} \quad r = \frac{5}{3}$

$|\frac{5}{3}| \geq 1$, sum does not exist

9. $2+\frac{6}{4}+\frac{18}{6}+\frac{54}{64}+\dots$

$\frac{6}{4} \Rightarrow 3 \quad \frac{6}{4} \cdot \frac{1}{2} = \frac{3}{4}$

$a_1 = 2 \quad r = \frac{3}{4}$

$S = \frac{2}{1-\frac{3}{4}} \Rightarrow \frac{2}{\frac{4}{4}-\frac{3}{4}} \Rightarrow \frac{2}{\frac{1}{4}} \Rightarrow 2 \cdot \frac{4}{1}$

$= \boxed{8}$

10. $-5-2-\frac{4}{5}-\frac{8}{25}-\dots$

$\frac{-2}{-5} = \frac{2}{5}$

$a_1 = -5 \quad r = \frac{2}{5}$

$S = \frac{-5}{1-\frac{2}{5}} \Rightarrow \frac{-5}{\frac{5}{5}-\frac{2}{5}} \Rightarrow \frac{-5}{3/5} \Rightarrow -5 \cdot \frac{5}{3}$

$= \boxed{-\frac{25}{3}}$

College Algebra – Chapter 11

Lesson 4

Write the repeating decimal as a fraction in simplest form.

11. 0.2222....

$0.22 + 0.0022 + 0.000022 \ldots$

$a_1 = .22 \quad r = \frac{.0022}{.22} \Rightarrow 0.01$

$S = \frac{.22}{1-.01} \Rightarrow \frac{.22}{.99} = \frac{22}{99} = \boxed{\frac{2}{9}}$

12. 0.444.....

$.44 + .0044 \ldots$

$a_1 = .44 \quad r = 0.1$

$S = \frac{.44}{1-0.1} = \frac{.44}{.99} = \boxed{\frac{4}{9}}$

13. 0.16161616.....

$.16 + 0.0016 + .000016$

$a_1 = 0.16 \quad r = \frac{.0016}{.16} \Rightarrow 0.1$

$S = \frac{.16}{1-.1} = \frac{.16}{.99} = \boxed{\frac{16}{99}}$

14. 0.625625625....

$.625 + .000625 \ldots$

$a_1 = .625 \quad r = \frac{.000625}{.625} \Rightarrow 0.001$

$S = \frac{.625}{1-.001} \Rightarrow \frac{.625}{.999} = \boxed{\frac{625}{999}}$

15. 32.32323232.....

$32 + .32 + .0032$

$a_1 = 32 \quad r = \frac{.32}{32} \Rightarrow 0.01$

$S = \frac{32}{1-0.1} = \frac{32}{.99} = \frac{3200}{99}$

$\boxed{32\frac{32}{99}}$

16. 130.130130130....

$130 + .130 + .000130$

$a_1 = 130 \quad r = 0.001$

$S = \frac{130}{1-0.001} \Rightarrow \frac{130}{.999} = \frac{130,000}{999}$

$\boxed{130\frac{130}{999}}$

College Algebra – Chapter 11
Lesson 5

Write the first six terms of the sequence.

1. $a_1 = 1;\ a_n = a_{n-1} + 31.$

$a_1 = 1$
$a_2 = 1+3 = 4$
$a_3 = 4+3 = 7$
$a_4 = 7+3 = 10$
$a_5 = 10+3 = 13$
$a_6 = 13+3 = 16$

2. $a_1 = 1;\ a_n = a_{n-1} - 5$

$a_1 = 1$
$a_2 = 1-5 = -4$
$a_3 = -4-5 = -9$
$a_4 = -9-5 = -14$
$a_5 = -14-5 = -19$
$a_6 = -19-5 = -24$

3. $f(0) = 4;\ f(n) = 2f(n-1).$

$f(0) = 4$
$f(1) = 2(4) = 8$
$f(2) = 2(8) = 16$
$f(3) = 2(16) = 32$
$f(4) = 2(32) = 64$
$f(5) = 2(64) = 128$

4. $f(0) = 10;\ f(n) = \frac{1}{2}f(n-1)$

$f(0) = 10$
$f(1) = \frac{1}{2}(10) = 5$
$f(2) = \frac{1}{2}(5) = \frac{5}{2}$
$f(3) = \frac{1}{2}\left(\frac{5}{2}\right) = \frac{5}{4}$
$f(4) = \frac{1}{2}\left(\frac{5}{4}\right) = \frac{5}{8}$
$f(5) = \frac{1}{2}\left(\frac{5}{8}\right) = \frac{5}{16}$

5. $a_1 = 2;\ a_n = (a_{n-1})^2 + 1$

$a_1 = 2$
$a_2 = (2)^2+1 \Rightarrow 4+1 = 5$
$a_3 = (5)^2+1 \Rightarrow 25+1 = 26$
$a_4 = (26)^2+1 \Rightarrow 676+1 = 677$
$a_5 = (677)^2+1 \Rightarrow 458,329+1 = 458330$
$a_6 = (458330)^2+1 \Rightarrow 210,066,388,901$

6. $a_1 = 1;\ a_n = (a_{n-1})^2 - 10$

$a_1 = 1$
$a_2 = (1)^2-10 \Rightarrow 1-10 = -9$
$a_3 = (-9)^2-10 \Rightarrow (81)-10 = 71$
$a_4 \in (71)^2-10 \Rightarrow 5041-10 = 5031$
$a_5 = 25,310,951$
$a_6 = 640,644,240,524,000$

Write a recursive rule for the sequence.

7. 21, 14, 7, 0, -7, ...

Common Diff: -7
$a_1 = 21 \quad d = -7$

$$a_n = a_{n-1} - 7$$

8. 54, 43, 32, 21, 10, ... Common Diff: -11

$a_1 = 54 \quad d = -11$

$$a_n = a_{n-1} - 11$$

9. 3, 12, 48, 192, 768, ...

Common Ratio: 4
$a_1 = 3 \quad r = 4$

$$a_n = 4a_{n-1}$$

10. 4, -12, 36, -108, ...

Common Ratio: -3
$a_1 = 4 \quad r = -3$

$$a_n = (-3)a_{n-1}$$

College Algebra – Chapter 11
Lesson 5

11. $44, 11, \frac{11}{4}, \frac{11}{16}, \frac{11}{64}, \ldots$

Common Ratio: $\frac{1}{4}$

$a_1 = 44 \quad r = \frac{1}{4}$

$a_n = \left(\frac{1}{4}\right)a_{n-1}$

12. $1, 8, 15, 22, 29, \ldots$

Common Diff: $+7$

$a_1 = 1 \quad r = 7$

$a_n = a_{n-1} + 7$

Write a recursive rule for the sequence.

13. $a_n = 3 + 4n$

$a_1 = 3 + 4(1)$
$= 3 + 4$
$= 7$

$a_1 = 7 \quad d = 4$

$a_n = a_{n-1} + 4$

14. $a_n = -2 - 8n$

$a_1 = -2 - 8(1)$
$= -2 - 8$
$= -10$

$a_1 = -10 \quad r = -8$

$a_n = a_{n-1} - 8$

15. $a_n = 12 - 10n$

$a_1 = 12 - 10(1)$
$= 12 - 10$
$= 2$

$a_1 = 2 \quad r = -10$

$a_n = a_{n-1} - 10$

16. $a_n = 9 - 5n$

$a_1 = 9 - 5(1)$
$= 9 - 5$
$= 4$

$a_1 = 4 \quad r = -5$

$a_n = a_{n-1} - 5$

17. $a_n = 12(11)^{n-1}$

$a_1 = 12 \quad r = 11$

$a_n = 11a_{n-1}$

18. $a_n = -7(6)^{n-1}$

$a_1 = -7 \quad r = 6$

$a_n = 6a_{n-1}$

Write an explicit rule for the sequence.

19. $a_1 = 3;\ a_n = a_{n-1} - 6.$

$a_1 = 3 \quad d = -6$

$a_n = 3 + (n-1)(-6)$
$= 3 - 6n + 6$
$= -6n + 9$

20. $a_1 = 16;\ a_n = a_{n-1} + 7$

$a_1 = 16 \quad d = 7$

$a_n = 16 + (n-1)(7)$
$= 16 + 7n - 7$
$= 7n + 9$

21. $a_1 = -2;\ a_n = 3a_{n-1}$

$a_1 = -2 \quad r = 3$

$a_n = -2(3)^{n-1}$

22. $a_1 = 13;\ a_n = 4a_{n-1}$

$a_1 = 13 \quad r = 4$

$a_n = 13(4)^{n-1}$

College Algebra – Chapter 11
Lesson 6, Day 1

Use the formula for P(n, r) to evaluate each expression.

1. P(6, 1)

$$\frac{6!}{(6-1)!} = \frac{6!}{5!} = \frac{6 \cdot 5!}{5!}$$

$$\boxed{6}$$

2. P(7, 3)

$$\frac{7!}{(7-3)!} \Rightarrow \frac{7!}{4!} \Rightarrow \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4!}$$

$$\boxed{210}$$

3. P(8, 2)

$$\frac{8!}{(8-2)!} \Rightarrow \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!}$$

$$= \boxed{20,160}$$

4. P(10, 6)

$$\frac{10!}{(10-6)!} \Rightarrow \frac{10!}{4!} \Rightarrow \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!}$$

$$\boxed{151,200}$$

5. P(9, 9)

$$\frac{9!}{(9-9)!} = \frac{9!}{0!} \Rightarrow \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$$

$$\boxed{362,880}$$

6. P(5, 5)

$$\frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$$

$$\boxed{120}$$

Use the formula for C(n, r) to evaluate each expression.

7. C(8, 3)

$$\frac{8!}{(8-3)! \cdot 3!} \Rightarrow \frac{8!}{5! 3!} \Rightarrow \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1} \rightarrow$$

$$\frac{336}{6} = \boxed{56}$$

8. C(7, 2)

$$\frac{7!}{(7-2)! 2!} \Rightarrow \frac{7!}{5! 2!} \Rightarrow \frac{7 \cdot 6 \cdot 5!}{5! \cdot 2 \cdot 1}$$

$$\frac{42}{2} = \boxed{21}$$

9. C(9, 4)

$$\frac{9!}{(9-4)! 4!} \Rightarrow \frac{9!}{5! 4!} \Rightarrow \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$\frac{3024}{24} = \boxed{126}$$

10. C(10, 5)

$$\frac{10!}{(10-5)! 5!} \Rightarrow \frac{10!}{5! 5!} \Rightarrow \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$\frac{30240}{120} = \boxed{252}$$

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11. $C(5, 5)$

$\frac{5!}{(5-5)!5!} \Rightarrow \frac{5!}{0!5!} = 1$

12. $C(6, 6)$

$\frac{6!}{(6-6)!6!} = \frac{6!}{0!6!} = 1$

Use the Fundamental Counting Principle to solve each problem.

13. How many different two letter codes can be made from the 26 capital letters of the alphabet if (a) repeated letters are allowed; (b) repeated letters are not allowed.

a) $26 \cdot 26 = 676$

b) $26 \cdot 25 = 650$

14. How many possible answer sheets are there for a ten-question true-false exam if no answer is left blank?

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{10} = 1024$

15. In how many ways can a president and vice president be chosen from an organization with 50 members?

$50 \cdot 49 = 2450$

16. How many four-digit numbers can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 if the first digit cannot be 0?

$9 \cdot 10 \cdot 10 \cdot 10 = 9{,}000$

College Algebra – Chapter 11
Lesson 6, Day 1

Use the formula for counting permutations to solve each problem.

17. In how many different ways can the members of a family of four be seated in a row of four chairs?

$4 \cdot 3 \cdot 2 \cdot 1 = 24$

18. Ten teams are entered in a bowling tournament. In how many ways can first, second, and third prizes be awarded?

$10 \cdot 9 \cdot 8 = 720$

19. Frederico has five shirts, three pairs of pants, and four ties that are appropriate for an interview. If he wards one of each type of apparel, how many different outfits can he wear?

$5 \cdot 3 \cdot 4 = 60$

20. A choreographer has to arrange eight pieces for a dance program. In how many ways can this be done?

$8! = 40,320$

College Algebra – Chapter 11
Lesson 6, Day 2

Use the formula for counting combinations to solve each problem.

1. A pizza menu offers pepperoni, peppers, mushrooms, sausage, and meatballs as extra toppings that can be added to your pizza. How many different pizzas can you order with two additional toppings?

$C(5,2)$ $\frac{5!}{(5-2)!\,2!} = \frac{5!}{3!\,2!} = \frac{5\cdot 4\cdot 3!}{3!\cdot 2\cdot 1} = \frac{20}{2} = 10$

2. In how many ways can 2 student representatives be chosen from a class of 15 students?

$C(15,2)$

$\frac{15!}{(15-2)!\,2!} \Rightarrow \frac{15!}{13!\,2!} = \frac{15\cdot 14\cdot 13!}{13!\cdot 2\cdot 1} \Rightarrow \frac{210}{2} = 105$

3. Students are allowed to choose 9 out of 11 problems to work for credit on an exam. How many ways can this be done?

$C(11,9)$ $\frac{11!}{(11-9)!\,9!} \Rightarrow \frac{11!}{2!\,9!} \Rightarrow \frac{11\cdot 10\cdot 9!}{2\cdot 1\cdot 9!} = \frac{110}{2} = 55$

4. You have to choose 3 out of 18 potential teammates to help you with a group assignment. How many ways can this be done?

$C(18,3)$ $\frac{18!}{(18-3)!\,3!} \Rightarrow \frac{18!}{15!\,3!} \Rightarrow \frac{18\cdot 17\cdot 16\cdot 15!}{15!\cdot 3\cdot 2\cdot 1} \Rightarrow \frac{4896}{6} = 816$

Solve the problem by any appropriate counting method.

5. How many ways can Ashley choose 4 out of 11 different canned goods to donate to charity?

$C(11,4) = \frac{11!}{(11-4)!\,4!} \Rightarrow \frac{11!}{7!\,4!} = \frac{11\cdot 10\cdot 9\cdot 8\cdot 7!}{7!\cdot 4\cdot 3\cdot 2\cdot 1} \Rightarrow \frac{7920}{24} = 330$

College Algebra – Chapter 11
Lesson 6, Day 2

6. How many ways can a computer store hire a salesclerk and a technician from seven applicants for the clerk position and four applicants for the technician position?

$7 \cdot 4 = \boxed{28}$

7. How many ways can six people be seated in a row with eight seats, leaving two seats vacant?

$C(8,6)$

$\frac{8!}{(8-6)!6!} \Rightarrow \frac{8!}{2!6!} \Rightarrow \frac{8 \cdot 7 \cdot 6!}{2 \cdot 1 \cdot 6!} \Rightarrow \frac{56}{2} = \boxed{28}$

8. Seven students arrive to rent two canoes and three kayaks. How many ways can two students be elected to rent the canoes and three to rent the kayaks?

$C(7,2) \Rightarrow \frac{7!}{(7-2)!2!} \Rightarrow \frac{7!}{5!2!} \Rightarrow \frac{7 \cdot 6 \cdot 5!}{5! \cdot 2 \cdot 1} = \frac{42}{2} = 21$

$C(5,3) \Rightarrow \frac{5!}{(5-3)!3!} \Rightarrow \frac{5!}{2!3!} \Rightarrow \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!} = \frac{20}{2} = 10$

$21 \cdot 10 = \boxed{210}$

9. Dakota is trying to decide in which order to visit five prospective colleges. How many ways can this be done?

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{120}$

10. How many ways can the first 4 numbers be called from the 75 possible bingo numbers?

$75 \cdot 74 \cdot 73 \cdot 72 \Rightarrow$

$\boxed{29,170,800}$